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Some Engineering Aspects of Ancient Structures

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Abstract: Large monumental constructions were a prominent feature in ancient Sri Lanka. Construction materials and techniques used in the past can be of significant interest to the modern engineer. The evolution of brick sizes during four ancient periods of Sri Lanka spanning from 375 B.C to 1350 A.D. was studied by using recorded data of bricks found on ancient construction sites. The calculated ratios and the relationships indicate that the length was relatively significant in the reduction of the brick sizes while breadth and thickness changed roughly in proportion to the length at a lower rate. The effect of ground condition, i.e. rock, strong soil and weak soil; and the effect of pedestals on a solid hemispherical dome type Stupa were analysed using SAP2000. It was found that a stupa, if unrestrained along its horizontal directions, could experience tension being developed at the centre when built on a weak soil. Also, when constructed on poor ground conditions a pedestal reduces the compressive stresses at the base. However, the pedestal causes higher hoop and radial tensile stresses closer to the top and bottom of the outer surface of the dome. Vaulted structures in Sri Lanka exhibit approximately similar span to wall thickness ratios, thus indicating the possibility of the design being governed by the geometry of the structure. Also the development of stresses in vaulted structures indicates that the maximum vertical stress is compressive at the base, while the maximum tensile stress is at the crown intrados.

Keywords: Brick sizes, Foundation, Hoop stresses, Pedestals, Vaulted structures

1. Introduction

1.1 Brick Sizes

Bricks were the most commonly used construction material in ancient constructions. Its sizes have evolved over the years. The lengths, breadths & thicknesses and hence the volume and the cross sectional area have undergone changes over time, with length, breadth and thickness changing from 19.8” to 8.2”, 10.3” to 4.74” and 3.4” to 1.55” respectively. There are claims suggesting the possibility of categorizing bricks and thereby the structures constructed using them into certain historical periods, depending on their dimensions [1], i.e. pre-Christian and post-Christian (0-300AD, 300-800AD and 800-1350AD). As for the dimensions of a brick, it was widely believed that the breadth and thickness of a brick were simple fractions of its length [1]. The methods of manufacturing as well as the reason for using the bricks were key factors for its reduction in size [2]

1.2 Stupas

Ancient stupas in Sri Lanka were built using bricks and this was structurally feasible since the maximum stresses developed in the dome were well below the strength of ancient bricks [3]. Since seismic effects are not of significant concern in Sri Lanka, considering only its self-weight provides a reasonable basis for analysis of stupas [3]. As for the results of stress analysis, the maximum vertical stresses are compressive, while the maximum hoop stresses are tensile [3].

1.3 Vaulted Structures

Vaulted structures, which are not as common as stupas, but found in image houses since the Polonnaruwa period, were also built using bricks. The vaults found in Sri Lanka could be broadly categorized under three types, namely; brick corbelled vaults, brick circular vaults and brick corbelled and circular vaults [2]. The performance of such masonry structures is governed by stability and hence a geometrical factor of safety is probably of relevance [4].

2. Study on Brick Sizes

2.1 Objective

This research sought to identify trends and relationships associated with the changes the brick sizes have undergone, based on the available data [1] over the four historic periods of Sri Lanka.

2.2 Methodology

The parameters considered in the analysis were, length (L), breadth (B), thickness (T), cross sectional area (BT), volume (V), length to breadth ratio (L/B), length to thickness ratio (L/T) and breadth to thickness ratio (B/T). The variations of these parameters with each other were studied [5].

2.3 Results and Discussion

As indicated in Table 1, the B vs L and T vs L relationships suggest that the reduction of the length significantly influenced the reduction of the breadth and thickness through the ages. Even more significant is the dependence of volume on both length and cross sectional area. However, Figures 1 and 2 show that the L/B and L/T ratios were not constant with changes of length. In other words length decreased more rapidly with time than breadth or thickness. This was perhaps the most interesting finding of the study; one that is not alluded to by Parker [1].

Table 1: Regression Coefficients

Quantities	R^2 Values
B vs L	0.659
T vs L	0.603
V vs L	0.905
V vs BT	0.943

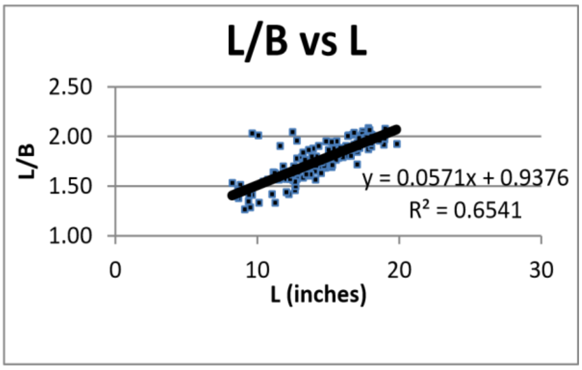

Figure 1: Variation of Length to Breadth ratio vs Length

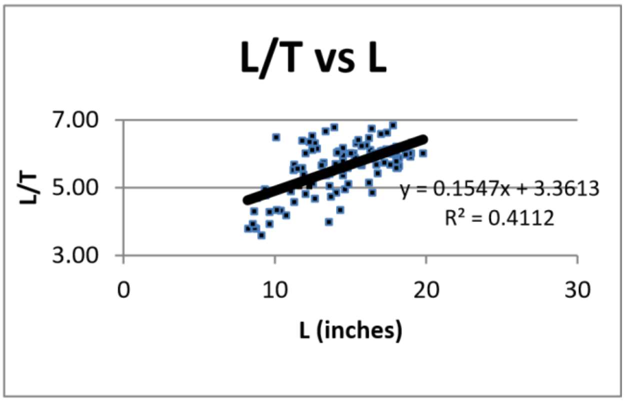

Figure 2: Variation of Length to Thickness ratio vs Length

3. Stupas

3.1 Objective

This component of the research identifies the effect of ground condition, i.e. rock, strong soil, and weak soil; and the effect of the pedestals on a solid hemispherical dome type Stupa. Although stupas were generally constructed from bedrock, modern construction of replicates may take place on less firm strata. The modelled Stupa was analysed using SAP2000 for the above cases to identify the nature and variation of vertical, radial and hoop stresses developed in the dome as well as the variation of the base pressure at the support level.

3.2 Methodology

The Stupa was modelled as a two dimensional axisymmetric structure, using the ASOLID element and then replicated to arrive at the final structure. The replicate angle as well as the material angle was taken as 8 degrees to ensure radial symmetry of the dome. Since it was the dome component that was of interest, the rest of the structure above it was modelled in such a way so that the geometric integrity of the structure remained intact. Furthermore, the dome was assumed to be hemispherical and hence, the actual height of Ruvanwelisaya [3] – a hemispherical stupa - was considered to be equal to the radius of the dome (i.e. 44 m). The stupa was meshed into squares of 3m x 3m and triangles at the curved edge, maintaining the aspect ratios within acceptable limits.

Springs were modelled to capture the effect of soil. The spring constants at supports were obtained by multiplying the modulus of subgrade reaction for the relevant ground condition by the tributary area of each support. Furthermore, to introduce coupling to the behaviour of the springs, the spring constant of the springs at the outer edge of the dome was doubled [6]. The moduli of subgrade reaction for rock, strong soil and weak soil were taken as 800,000 kN/m^3, 80,000 kN/m^3 and 8,000 kN/m^3 respectively.

Initially, the dome was analysed while restraining it only vertically, i.e., either fixed at supports or on springs only in the vertical direction. Next, to consider the friction effect of the soil, it was restrained in the two horizontal directions (radial and tangential), with the aforementioned restraint conditions for the vertical direction.

3.3 Results and Discussion

When unrestrained in the two horizontal directions, thus neglecting the frictional effect of the underlying soil, compressive stresses increase in radial, vertical and tangential directions as the ground conditions vary from rock to weak soil. Furthermore, zones of tensile stress move inwards from the outer surface of the dome to the centre for radial and hoop stresses and the numerical values of the stresses also increase (see Figure 3). This creates an undesirable situation, as tensile stresses in the centre could weaken the structure considerably.

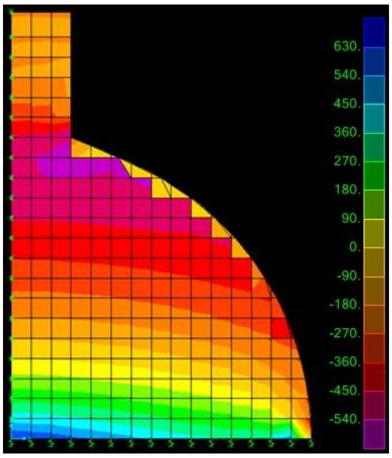

Figure 3: Hoop Stress Variation for Weak Soil Condition (Unrestrained in the Two Horizontal Directions)

However, for more realistic site conditions, the effect of the underlying soil friction should also be accounted for in the analysis. A frictional shear force equal to (tanφ x F_Z) + (c x Tributary area) will be applied by the soil. For this analysis, a friction angle (φ) of 26^{o}and a cohesion (c) of 0 kN/m^2 was used to represent the lower bound for a purely cohesionless soil; while a friction angle (φ) of 0^o and a cohesion (c) of 100 kN/m^2 used to represent the upper bound for a purely cohesive soil(Note: the actual adhesion between soil and foundation would be less than this as well).

The results for the first case indicated that the frictional shear force in the radial direction is greater than the reaction in that direction at each support joint. Therefore, this indicates that the surrounding soil prevents the movement of the structure in the radial direction. Hence, for a purely cohesionless soil, restraining the structure in the two horizontal directions at the support joints would provide a more accurate approximation of the actual state.

However, results for the second case indicated that the frictional shear force in the radial direction is less than the reaction in that direction, even in a very cohesive soil. Hence, stupas on purely cohesive soils may develop internal tension.

When restrained in the two horizontal directions, thus representing the true condition for most cases, tensile hoop stresses are greater than the tensile radial stresses. Maximum tensile radial stresses occur closer to the base, while there are two identifiable tensile zones for hoop stresses; one adjacent to the base and the other adjacent to the top (see Figure 4).

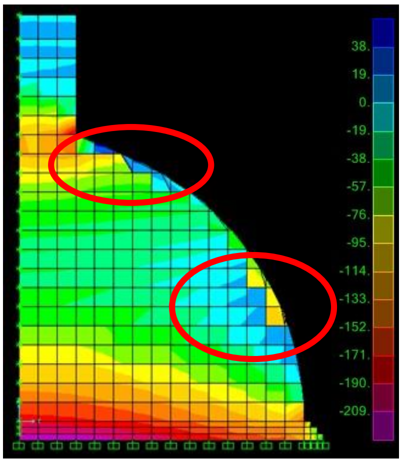

Figure 4: Hoop Stress Variation for Fixed Base Condition (Restrained in the Two Horizontal Directions)

As illustrated in Figure 5, as the foundation condition weakens the maximum hoop tensile stress at the zone closer to the top increases regardless of the presence of the pedestal. However, the stresses with the pedestals are marginally higher than when without pedestals. This may just be because of the additional load from the pedestal.

As far as base pressure is concerned, for the fixed base case, as illustrated in Figure 6, the relatively higher stresses for the case with the pedestals could be due to the added weight of the pedestals. The increase in pressure at the outer edge may be due to the modelling technique of doubling the end spring stiffness. Apart from that, the pressures decrease from centre to edge, somewhat reflecting the weight of the brickwork directly above each point.

However, as illustrated in Figure 7, when on weak soil, once again apart from the outer edge, the pressures from centre to edge are very uniform. This may be because stupas on weak soils display more arch action, creating greater pressure close to the edges and matching those at the centre due to the loads from greater brick heights. Also, the gap between the two lines is much less compared to the earlier case.

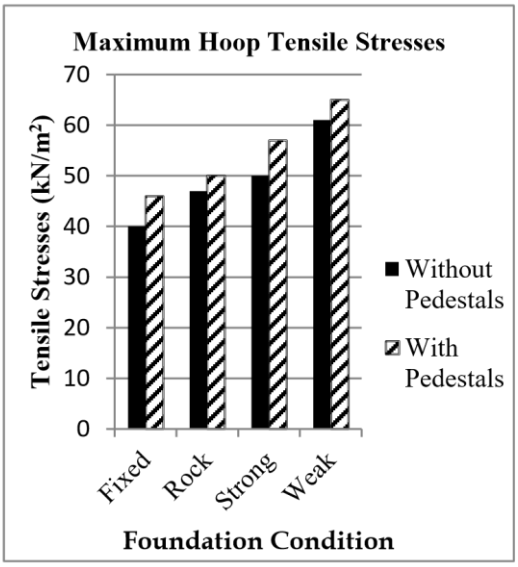

Figure 5: Maximum Hoop Tensile Stresses for Different Foundation Conditions

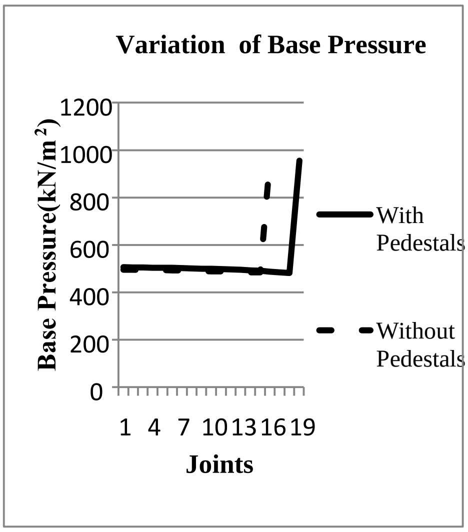

Figure 7: Variation of Base Pressure for Weak Soil Condition

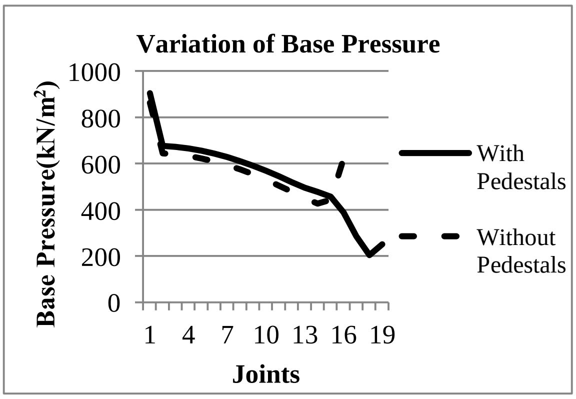

Figure 6: Variation of Base Pressure for Fixed Base Condition

4 Vaulted Structures

4.1 Objective

Since the geometry of the structure is considered critical in arches and vaulted structures, this research looks to identify combinations of spans and wall thicknesses of existing vaulted structures in Sri Lanka. In addition, these were modelled using SAP 2000 to identify the nature and variation of the vertical and horizontal stresses throughout the vault and the variation of the base pressure at the support levels.

4.2 Methodology

The brick masonry vaulted structures at the Thivanka Image House and the Thuparama Image House were considered for the analysis. The dimensions of the former were obtained from the drawings at the Polonnaruwa Branch of the Central Cultural Fund, while the dimensions of the latter were measured at site due to the absence of prerecorded data (see Figures 8 and 9).

A cross section of the vault was modelled in SAP2000, using a plane stress element with 1 m thickness. The structure was meshed into squares, trapeziums and triangles while maintaining the aspect ratios within acceptable limits. Since the drawings for the Thivanka Image House indicated that the structure was placed on a rock foundation at a depth of 1m from the ground level, the support conditions were assigned as fixed.

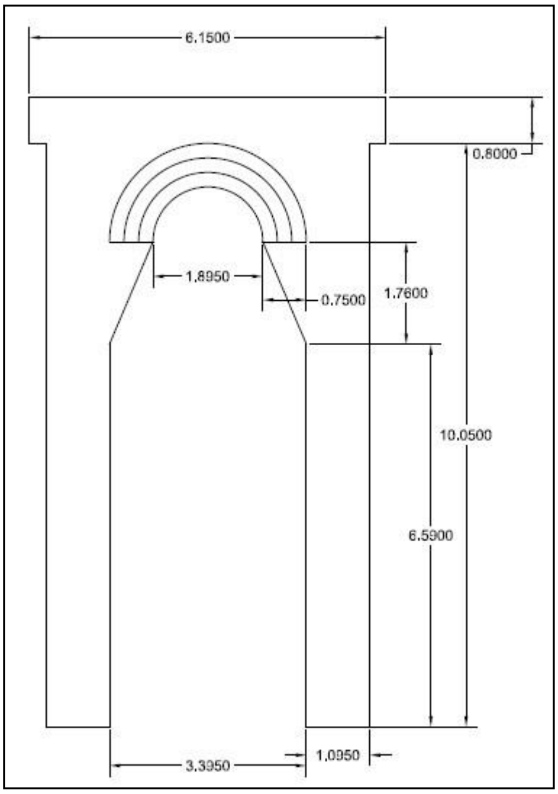

Figure 8: Dimensions of Thivanaka Image House (in meters)

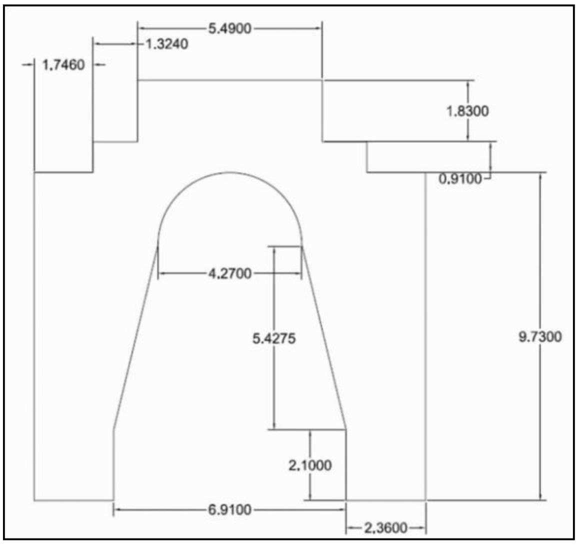

Figure 9: Dimensions of Thuparama Image House (in meters**)**

4.3 Results and Discussion

Table 2 indicates the span to wall thickness ratios that were observed. The two values for the Thivanka image house are from two areas, in the larger one of which only the springings of the arch are visible. The ratios are virtually identical and close to 3, suggesting that it could have been a geometrical design rule.

Table 2: Geometrical Relationships of Thivanka and Thuparama Image Houses

Location	Span (m)	Wall thickness (m)	Span/ Wall thickness ratio
Thivanka-1	3.4	1.12	3.04
Thivanka-2	7.96	3	2.65
Thuparama	6.91	2.3	3.00

As far as stresses are concerned, the maximum tensile stresses in both vertical and horizontal directions occur at the crown intrados of the vault. This could be attributed to the deformed shape under the self-weight of the structure. Furthermore, a small principal tensile stress is developed throughout the entire structure due to the relative magnitudes and directions of the vertical and horizontal stresses.

As for compressive stresses, the higher values at the base would be due to the effect of the selfweight. The horizontal restraint causes the vertical compressive stresses to increase when moving outwards from the structure intrados at base level (Figure 10); but there is an opposite trend at intermediate levels. The deformed shape from the modelling in ref. [5] gives the impression of arch walls bulging outwards due to vertical load, but restrained along the base.

Furthermore, maximum vertical stresses are compressive at the base, while the maximum tensile stresses are at the crown intrados (Figure 11).

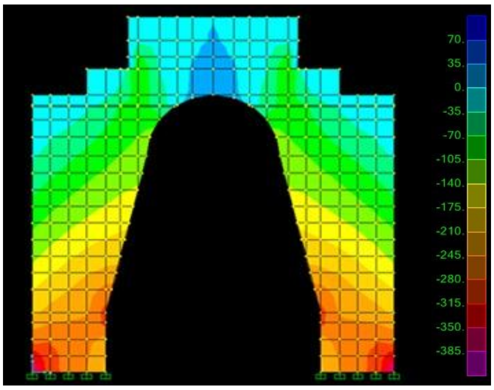

Figure 10: Variation of Vertical Stress in the Thuparama Image House

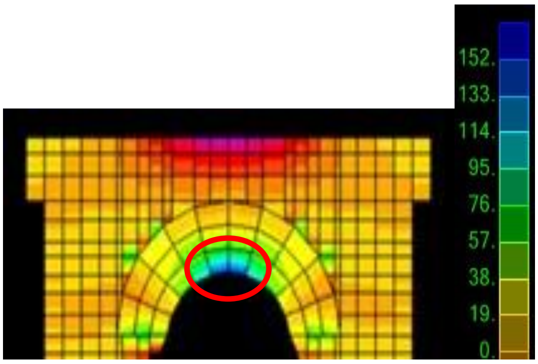

Figure 11: Tensile Zone Due to Horizontal Stresses in the Vault

Table 3: Comparison of Compressive and Tensile Stresses in Stupa and Vaulted Structures

`	Vertical Compressive Stress at Base(kN/m²)		Maximum Tensile Stress(kN/m²)		
	Maximum	Range	Value	Direction	Location
Stupa	675	46-675	98-	Hoop	Surface
Vault 1	340	207-340	147	Crown, Horizontal	intrados
Vault 2	395	205-395	25.5	Crown, Horizontal	intrados

As indicated in Table 3, the compressive stresses induced in the stupa (fixed base) are much greater than that of the vaulted structures. The much larger self-weight of the Stupa would be the reason for this. Furthermore, the range of variation of compressive stress for the stupa is much greater when compared to the vaults. This could be due to the shape and hence the variation of the mass in the dome of the Stupa.

Table 3 also shows that two distinct vaulted structures with significant variations in dimension, yield somewhat similar variations in stresses, perhaps because they had approximately similar span to wall thickness ratios.

Furthermore, both compressive and tensile stresses are within the strength limits of modern brick work which are around 1.5 N/mm^2 and 0.2 N/mm^2 respectively [7]. However, the likelihood of a tensile failure is greater in the vault.

5. Conclusions

5.1 Brick Sizes

Although reduction of the length significantly influenced the reduction of the breadth and thickness through the ages, length decreased more rapidly with time than breadth or thickness.

5.2 Stupas

The properties of the sub soil, the friction angle and the cohesion in particular, is of significant interest, in designing and constructing a stupa, because a lower frictional shear force could induce undesirable tensile stresses at the centre, when constructed on weak soil.

Stupas on weak soils have more uniform pressure distributions under their bases, while those on rock would have higher pressures at the centre.

The effects of varying ground conditions are much more significant than the variation between a stupa with and without a pedestal.

5.3 Vaulted Structures

Approximately similar span to wall thickness ratios for differing vault spans suggest that a geometrical guideline may have been used in their design. This is confirmed by the fact that two distinct vaulted structures with significant variations in dimension, yield similar variations in stresses for approximately similar span to wall thickness ratios.

Vertical stresses are maximum and compressive at the base, while tensile stresses are maximum at the crown intrados.

Compared to the stupa, the compressive stresses in the vaults are smaller. Compressive and tensile stresses for both the stupa and vaults are less than the characteristic strengths of modern brickwork; however, the likelihood of a tensile failure is greater in the vault.

Acknowledgement

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