SECM/15/35

Construction of Consistent Mass Spring Model Based on Meta-Modeling Theory for Seismic Response Analysis of Large Scale Bridge Structures

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Abstract: Meta-modeling theory is aimed at constructing a set of analysis models which are consistent with continuum mechanics or a solid element model. This paper presents a consistent mass spring model (CMSM) of a large scale bridge structure, which is constructed according to the meta-modeling theory to make efficient seismic response analysis. The CMSM shares the same dynamic characteristics as the solid element model and can be used to study fundamental seismic responses for a complicated large scale bridge structure that consists of piers and decks. In the numerical experiment, time history analysis is made for six different bridge structures. Full comparison is made for a CMSM and a solid element model of these six bridge structures, and it is shown that the CMSM is able to estimate the dynamic responses such as displacement and base shear for a certain class of ground motions.

Keywords: bridge structure, consistent modeling, continuum mechanics, mass spring model, structural mechanics.

1. Introduction

Construction of a few different fidelity models at the beginning of complex structure's analysis [1, 2] is a common practice among present engineers. If a numerical model of desired fidelity could be constructed for a target structure, we can choose a suitable analysis method and operate numerical simulation that uses the analysis model and the analysis method. Such model selection is very important especially for a dynamic seismic analysis of large structure which requires larger numerical computational resources as compared to a quasistatic analysis of small structure.

The fidelity of the model and accuracy of analysis are directly related which means that the highest fidelity model should have highest accuracy among a set of models which is developed for a particular structure. However, different fidelity models ought to share the fundamental dynamic characteristics such as natural frequency. It is meaningless to compare seismic responses of models which have different fundamental dynamic characters.

Mass spring model is popular on account of its simplicity and conservative response predictions, see references [3, 4, 5, 6]. For typical mass spring model, the target structure is discretized with set of beam elements. The lumped mass for each node of spring mass model is estimated from the portion of the weight of target structure, which is called "tributary area consideration". There are mainly two ways to estimate stiffness of spring in typical mass spring model, which are; static and geometric methods [3, 7]. The static method uses an arbitrary static load applied to a single layer of the full (3D) finite element model, it works as a pushover analysis [8, 9], while the geometric method considers the geometric shape of the cross-section to calculate sectional moment of inertia and shearcoefficients.

An issue with the ordinary mass spring model discussed above is that it does not consider the consistency with other more sophisticated models. It is easy to reproduce observed or synthesized dynamic response by tuning of mass spring model's parameters (mass and stiffness) for particular input motion but it may not be applicable for other input motions. However, construction of a consistent mass spring model; the one having same fundamental dynamic characteristics as more sophisticated models, is surely desirable.

The authors are proposing meta-modeling theory [10, 11, 12, 13, 14, 15], which allocates structural mechanics as mathematical approximation of solving a Lagrangian problem of continuum mechanics. The key concept of meta-modeling is that the same physical problem of continuum mechanics is solved by all modelings using distinct mathematical approximations. Therefore, it is well expected to construct a mass spring model of the same fundamental dynamic characteristics as a continuum mechanics model, according to the metamodeling theory.

This paper aims to apply meta-modeling based consistent mass spring model (CMSM) for fundamental seismic response analysis of six bridge structures with different pier arrangements. The contents of this paper are as follows. In Section 2, meta-modeling is briefly explained and in Section 3, the approximations made for deriving the governing equations of the CMSM from continuum mechanics theory are presented. In Section 4, we carry out the numerical experiment to obtain the fundamental seismic response for six multi-span bridge structures by employing CMSMs. Concluding remarks are made at the end in section 5.

2. Meta-modeling theory

In the meta-modeling theory, modeling means to create a mathematical problem for a target physical problem. There are many ways to develop a distinct mathematical problem, depending on the accuracy that is expected in solving the physical problem. The meta-modeling theory delivers a set of consistent modelings which produce an approximate solution of the original modeling. As an example in structural problems, the meta-modeling theory uses continuum mechanics modeling as the basic modeling. Some of the structure mechanics modelings are specified as consistent modelings of the continuum mechanics modeling. Then, those consistent structure mechanics modelings produce an approximate solution of the continuum mechanics modeling.

For simplicity, we assume small deformation, dynamic state, and linear isotropic elasticity. A boundary value problem of solid continuum mechanics is converted to a variational problem of using a Lagrangian,

$$\mathcal{L}[\mathbf{v},\boldsymbol{\epsilon}] = \int_V \frac{1}{2}\rho\mathbf{v}\cdot\mathbf{v} - \frac{1}{2}\boldsymbol{\epsilon}:\mathbf{c}:\boldsymbol{\epsilon}\,\mathrm{d}v, \quad (1)$$

where $\mathbf{v}$ is velocity, ρ is density, $\boldsymbol{\epsilon}$ is strain tensor and $\mathbf{c}$ is elasticity tensor; for a given displacement vector, $\mathbf{u}$, $\boldsymbol{\epsilon}$ is computed as $\boldsymbol{\epsilon} = sym\{\nabla\mathbf{u}\}$ where sym stands for the symmetric part of the secondorder tensor of $\nabla\mathbf{u}$ and ∇ stands for spatial differentiation operator; and V is the analysis domain.

In structural mechanics, the integral of $\frac{1}{2}\boldsymbol{\epsilon}:\mathbf{c}:\boldsymbol{\epsilon}$, (i.e., strain energy density) is replaced by, say, $\frac{1}{2}E\epsilon^2$, for bar theory [16], where ϵ is a normal strain component and E is Young's modulus. This strain energy density corresponds to a stress-strain relation of $\sigma = E\epsilon$, where σ is normal stress component in the same direction as ϵ. However, for this stress strain relation to hold, normal strain components in the transverse directions are non-zero. Therefore, $\sigma = E\epsilon$ is often regarded as an assumption of one dimensional stress-strain relation. It is not acceptable to make an assumption which is not experimentally validated. As mentioned, $\sigma = E\epsilon$ holds when transverse normal strain components are non-zero, but the presence of these components are ignored. Meta-modeling replaces the integrand and changes the Lagrangian in the following form:

$$\mathcal{L}^*[\mathbf{v},\boldsymbol{\epsilon},\boldsymbol{\sigma}] = \int_V \frac{1}{2}\rho\mathbf{v}\cdot\mathbf{v} - \left(\boldsymbol{\sigma}:\boldsymbol{\epsilon} - \frac{1}{2}\boldsymbol{\sigma}:\mathbf{c}^{-1}:\boldsymbol{\sigma}\right)\mathrm{d}v, \quad (2)$$

where $\boldsymbol{\sigma}$ is stress tensor and $\mathbf{c}^{-1}$ is the inverse of $\mathbf{c}$. By selecting σ as a unique non-zero component of $\boldsymbol{\sigma}$, without making any assumption, we can derive $\sigma = E\epsilon$ from $\delta\mathcal{L}^* = 0$ with respect to $\boldsymbol{\sigma}$ (or σ) for quasi-static state.

The use of the Lagrangian of Eq. (2) is the basic concept of meta-modeling. The governing equations of beam theory and plate theory are derived from this $\mathcal{L}^*$, just by using a suitable subset of the function space of $\{\mathbf{u},\}$, from which the arguments of $\mathcal{L}^*$, i.e., $\boldsymbol{\epsilon}$ and $\boldsymbol{\sigma}$, are computed. We have to emphasize that there is no need to make a physical assumption of $\sigma = E\epsilon$, (which is not experimentally validated), in deriving the governing equations. We regard using a subset of $\{\mathbf{u}, \boldsymbol{\sigma}\}$ to solve $\delta\mathcal{L}^* = 0$ as mathematical approximation.

3. Consistent mass spring model

A mass spring model is a set of masses and linear springs, and the direction of the mass movement is fixed. As the simplest case, we study a mass spring model which consists of two masses. We seek to construct a mass spring model which shares the same fundamental dynamic characteristics as continuum mechanics; this model is called consistent mass spring model (CMSM).

According to the meta-modeling theory, we consider an approximate displacement function of the following form:

$$(\mathbf{x},t)=\sum_{\alpha=1}^{2}U(t)\boldsymbol{\phi}^{\alpha}(\mathbf{x}), \qquad (3)$$

where U^{α} is displacement of the α-th mass point and $\boldsymbol{\phi}^{\alpha}$ is the corresponding displacement mode. By definition, the displacement is required to satisfy the following two requirements:

A1) $(\mathbf{x}^{\alpha})$ is a unit vector.

A2) $(\mathbf{x}^{\beta})$ vanishes for $\alpha\neq\beta$.

Here, $\mathbf{x}^{\alpha}$ is the location of the α-th mass point. We substitute Eq. (3) into Eq. (1), and obtain

$$\mathcal{L}=\sum_{\alpha,\beta=1}^{2}\frac{1}{2}m^{\alpha\beta}\dot{U}^{\alpha}\dot{U}^{\beta}-\frac{1}{2}k^{\alpha\beta}U^{\alpha}U^{\beta}, \qquad (4)$$

where

$$m^{\alpha\beta}=\int_V\rho\boldsymbol{\phi}^{\alpha}\cdot\boldsymbol{\phi}^{\beta}\,\mathrm{d}v,\qquad k^{\alpha\beta}=\int_V\nabla\boldsymbol{\phi}^{\alpha}:\mathbf{c}:\nabla\boldsymbol{\phi}^{\beta}\,\mathrm{d}v. \qquad (5)$$

Since a Lagrangian of a conventional mass spring model of two masses is in the form of

$$\frac{1}{2}(\cdot)(\dot{U}^1)^2+\frac{1}{2}(\cdot)(\dot{U}^2)^2-\frac{1}{2}(\cdot)(U^2-U^1)^2-\frac{1}{2}(\cdot)(U^2)^2$$

with $(\cdot)$ being a suitable scalar, $\mathcal{L}$ of Eq. (4) becomes the above, if the following two requirements are satisfied:

B1) $m^{12}=0$.

B2) $k^{12}+k^{22}=0$.

It is readily seen that finding two functions $\boldsymbol{\phi}^1$ and $\boldsymbol{\phi}^2$ which satisfy the four conditions of A1, A2, B1 and B2 is generally not possible.

Now we can consider dynamic mode shapes to construct a mass spring model, so that it shares the same dynamic fundamental characteristics with a continuum model. We suppose that two dynamic modes $\{\boldsymbol{\psi}^{\alpha},\omega^{\alpha}\}$ ($\alpha=1$ or 2), are given; $\boldsymbol{\psi}^{\alpha}$ is a mode shape and ω^{α} is a natural frequency. Recall that the dynamic mode satisfies

$$(\omega^{\alpha})^2\boldsymbol{\psi}^{\alpha}+\nabla\cdot(\mathbf{c}:\nabla\boldsymbol{\psi}^{\alpha})=0, \qquad (6)$$

and

$$\int_V\rho\boldsymbol{\psi}^{\alpha}\boldsymbol{\psi}^{\beta}\,\mathrm{d}v=0,\quad\int_V\nabla\boldsymbol{\psi}^{\alpha}:\mathbf{c}:\nabla\boldsymbol{\psi}^{\beta}\,\mathrm{d}v=0, \qquad (7)$$

for $\alpha\neq\beta$.

We can use Eq. (1) or (2) but for simplicity, we use $\mathcal{L}$ of Eq. (1), and, substituting $\mathbf{u}=\sum u^{\alpha}\boldsymbol{\psi}^{\alpha}$ into it, we obtain

$$\mathcal{L}=\sum_{\alpha,\beta=1}^{2}\frac{1}{2}m^{\alpha}(\dot{U}^{\alpha})^2-\frac{1}{2}k^{\alpha}(U^{\alpha})^2 \qquad (8)$$

where

$$m^{\alpha}=\int_V\rho\boldsymbol{\psi}^{\alpha}\cdot\boldsymbol{\psi}^{\alpha}\,\mathrm{d}v,\qquad k^{\alpha}=\int_V\nabla\boldsymbol{\psi}^{\alpha}:\mathbf{c}:\nabla\boldsymbol{\psi}^{\alpha}\,\mathrm{d}v. \qquad (9)$$

Due to the orthogonality, Eq. (7), $\{\boldsymbol{\psi}^{\alpha}\}$ does not produce cross terms. Furthermore, due to Eq. (6), it is readily seen that m^{α} and k^{α} of Eq. (9) satisfy

$$(\omega^{\alpha})^2m^{\alpha}=k^{\alpha}, \qquad (10)$$

for $\alpha=1$ and 2.

Now, we seek to find suitable linear combinations of $\{\boldsymbol{\psi}^{\alpha}\}$ that satisfy the requirements A1 and A2. To this end, we consider the following combination:

$$\boldsymbol{\phi}^{\alpha}=\sum t^{\alpha\beta}\boldsymbol{\psi}^{\alpha}, \qquad (11)$$

where $t^{\alpha\beta}$ is a component of two-by-two matrix. It is readily seen that this matrix can be determined when $\boldsymbol{\psi}^1$ and $\boldsymbol{\psi}^2$ do not change the direction and are parallel to each other.

4. CMSM for bridge structures

4.1 Problem setting

As a more realistic example, a CMSM is constructed for a set of multi-span curved and straight continuous bridge structures. Three curved and three straight bridge structures with different types of pier arrangement are studied; see Figure 1. The longitudinal and transverse directions are considered separately in this numerical study. The CMSM for the transverse direction includes two dynamic modes while that for the longitudinal direction uses only first mode. This is because in the longitudinal direction, the first mode has much lower natural frequency than other modes.

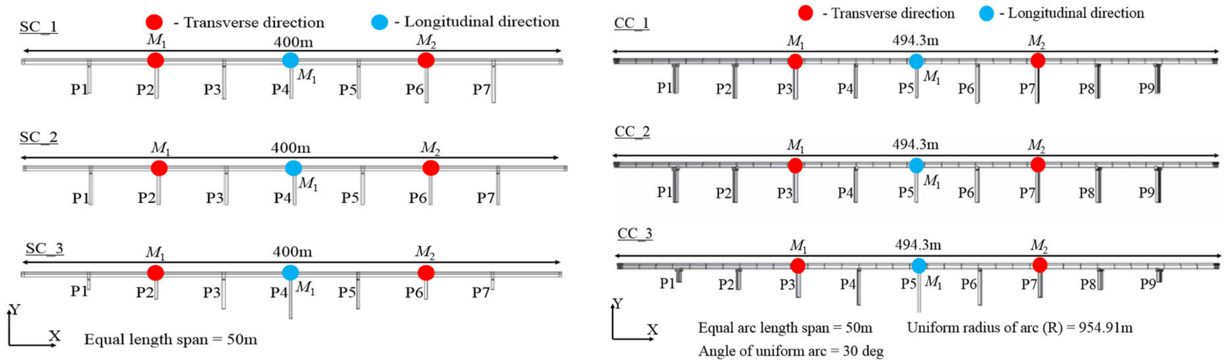
A schematic view of the CMSM with the third spring that connects the top mass to the ground is shown in Figure 2. The stiffness values, K_1, K_2 and K_3, are computed as follows:

$$K_1 = k_{22} + k_{12},\ K_2 = -k_{12},$$

$$\text{and}\ \ K_3 = k^{11} + k^{12}.$$

Tie connection is used for the connection between the pier and the deck in this problem. This is the simplest connection, and more sophisticated connection could be used if more detailed information is available for the connection. Table 1 shows the material properties of both the pier and the deck. Linear isotropic elasticity is assumed. The ground motion displayed in Figure 3 is employed. frequency domain.

First, we construct a solid element model, in order to obtain first two dynamic mode shapes in the transverse direction ($\boldsymbol{\psi}^1{}_z$ and $\boldsymbol{\psi}^2{}_z$) and first dynamic mode shape in the longitudinal direction ($\boldsymbol{\psi}_x^1$); see Figure 4(a) and 5(a) for $\boldsymbol{\psi}^1{}_z$ and $\boldsymbol{\psi}^2{}_z$ of cases SC_1 and CC_1 respectively. Approximate displacement functions ($\boldsymbol{\phi}^1{}_z$ and $\boldsymbol{\phi}^2{}_z$) of cases SC_1 and CC_1 for transverse direction are shown in Figure 4(b) and 5(b) respectively. Second, we determine locations of



(a) (b)

Figure 1: Geometric and mass points' information about multi-span bridge structures: (a) straight continuous (SC); and (b) curved continuous (CC).

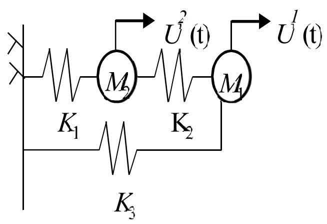
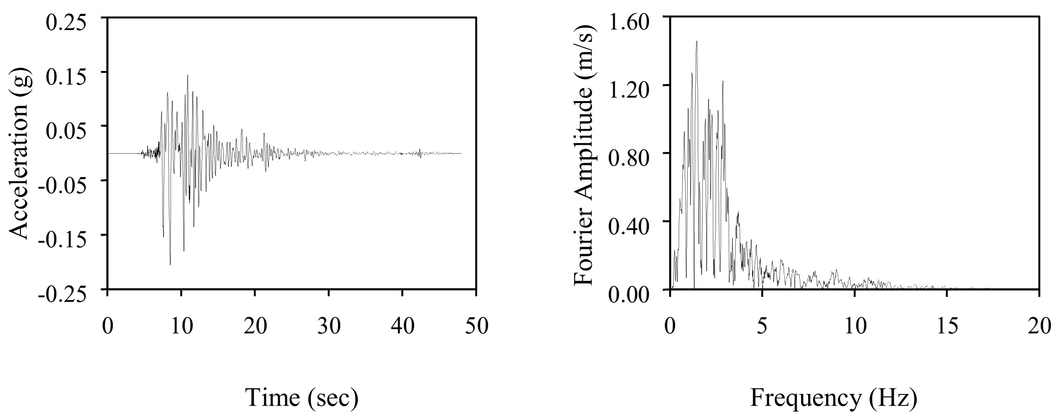


Figure 2: Schematic view of a consistent mass spring system consisting of two mass points.

Table 1: Material data of multi- span bridge structures (SC & CC).

Item	E / GPa	ρ / Kgm^{-3}	v
Pier	24	2400	0.2
Deck	200	2000	0.3



(a) (b)

Figure 3: Input ground motion for multi-span bridge structure: (a) in time domain; and (b) in frequency domain

mass points along the deck axis, considering target locations of response output from the models; see Figure 1. Third, CMSM parameters are computed from the dynamic mode shapes and the mass points' location.

4.2 Results and discussion

Natural frequencies of the CMSMs in the transverse and longitudinal directions are presented in Tables 2 and 3, respectively; the natural frequencies of the original solid element models are presented, too. As is seen, the natural frequencies of the CMSMs coincide with those of the solid element models.

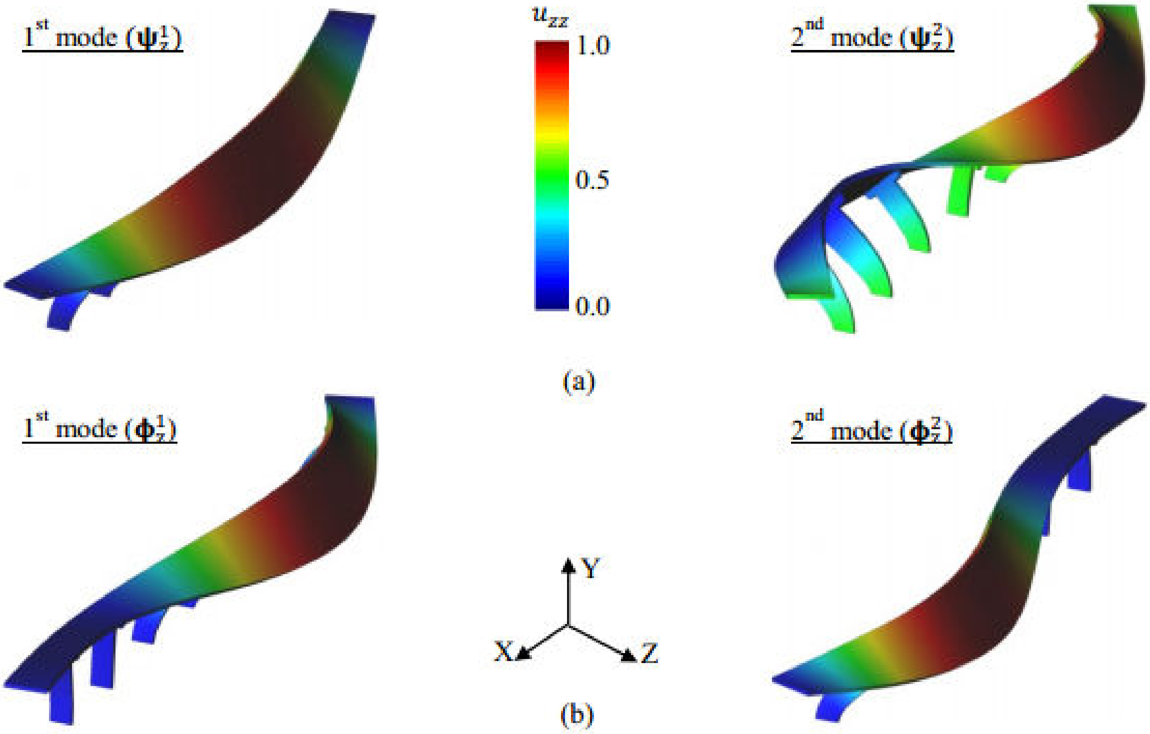


Figure 4: Solid bridge model along the transverse direction of bridge (SC_1): (a) first two dynamic modes; and (b) developed approximate displacement modes.

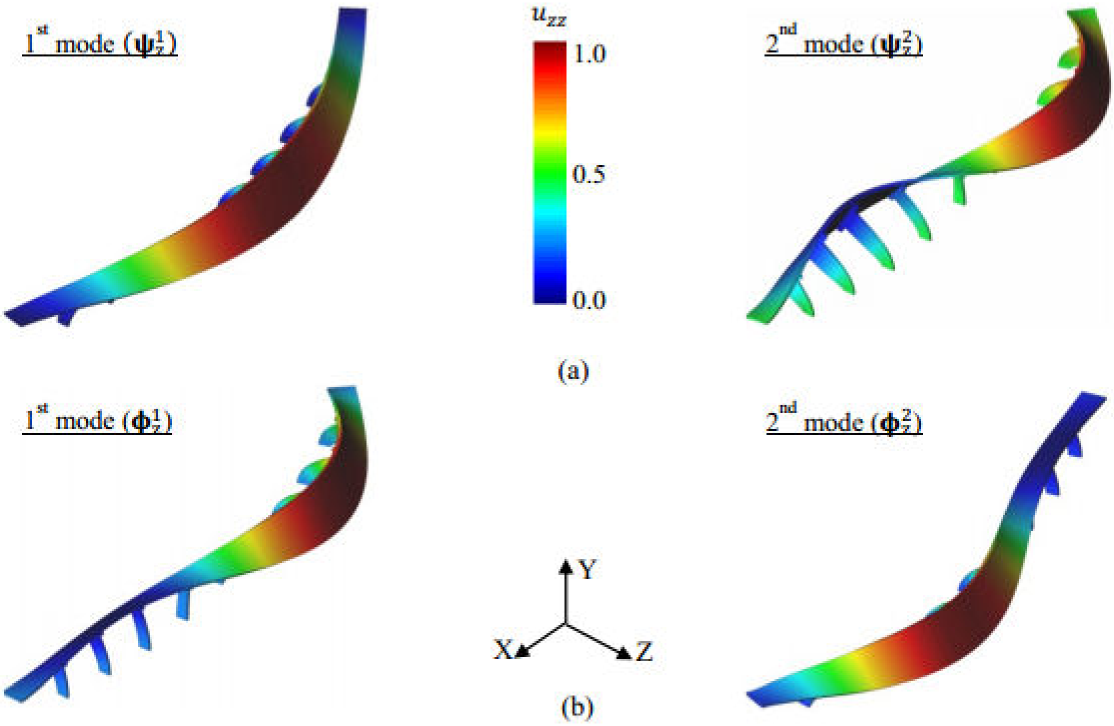


Figure 5: Solid bridge model along the transverse direction of bridge (CC_1): (a) first two dynamic modes; and (b) developed approximate displacement modes.

Table 2: Natural frequencies along the transverse direction of multi-span bridge structure (CMSM and solid element model)

Case ID	Frequency / (Hz)				Difference / (%)	
	CMSM		Solid			
	1st mode	2nd mode	1st mode	2nd mode	1st mode	2nd mode
SC_1	1.625	2.610	1.624	2.610	0.062	0.000
SC_2	1.752	2.703	1.752	2.702	0.000	0.037
SC_3	2.191	3.882	2.190	3.881	0.046	0.026
CC_1	1.541	2.152	1.540	2.151	0.065	0.046
CC_2	1.692	2.254	1.692	2.253	0.000	0.044
CC_3	1.894	3.131	1.893	3.130	0.053	0.032

Table 3: Natural frequencies along the longitudinal direction of multi-span bridge structure (CMSM and solid element model)

Case ID	Frequency / (Hz)		Difference / (%)
	CMSM	Solid (1st mode)	
SC_1	0.630	0.630	0.000
SC_2	0.680	0.680	0.000
SC_3	1.640	1.640	0.000
CC_1	0.713	0.712	0.140
CC_2	0.730	0.730	0.000
CC_3	1.624	1.623	0.062

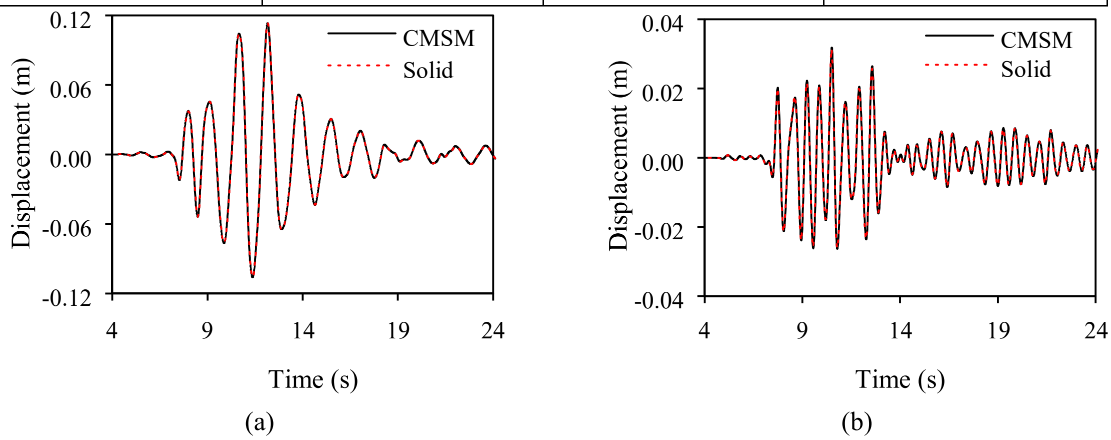


Figure 6: The displacement results of SC_1 at M_I mass point location: (a) the CMSM for the longitudinal direction (z-direction); and (b) the CMSM for the transverse direction (x-direction).

Time series of displacement responses of the CMSM is compared with that of the original solid element model; see Figure 6 for case SC_1. It is seen that the response of the CMSM matches well with that of the solid element model. Relative errors of the maximum displacement in the longitudinal and transverse directions of the CMSM are presented in Tables 4(a) and 4(b), respectively. As is seen, the maximum relative error in all the cases is 2.852%

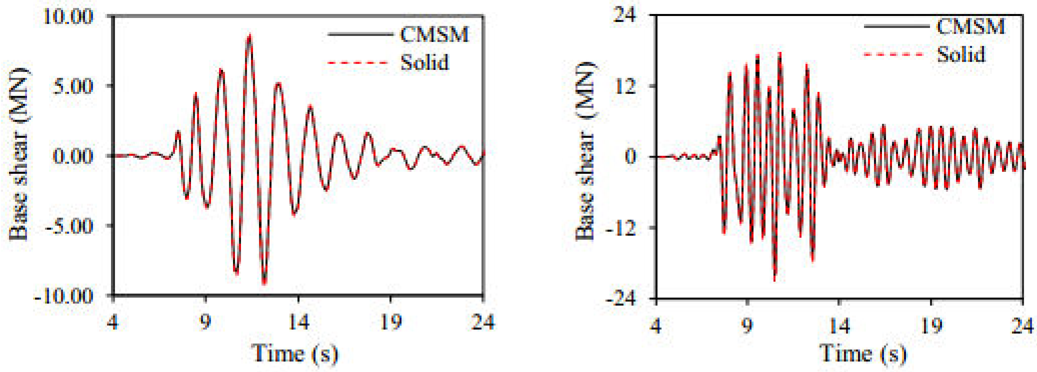
Next, shear force values at the base of the fourth pier (P4) are estimated; see Figure 1 for the location of P4. The cross sectional shear force is computed as follows:

$$\mathsf{F}(t) = \sum u^{\alpha}(t) \int \mathbf{n} \cdot (\mathbf{c}(\mathbf{0}) : \nabla\boldsymbol{\psi}^{\alpha}(\mathbf{0}))\mathrm{d}s, \qquad (6)$$

where $\mathbf{x} = \mathbf{0}$ corresponds to the base of the target pier, $\mathbf{n}$ is the unit normal on base, and the surface integration is made on the base. This computation is logical in the sense that the present CMSM is essentially the same as the modal analysis [5, 6, 17], and is able to compute local responses by using the approximate displacement, i.e., $\mathbf{u} = \sum u^{\alpha}\boldsymbol{\psi}^{\alpha}$.

In Figure 7, **F** is presented for the longitudinal and transverse directions. Relative errors of the maximum resultant force are summarized in Tables 5(a) and 5(b). As is seen, the maximum relative error in all the cases is 3.524%, and it is clear that the CMSM can be used to approximately estimate structural seismic responses for certain class of ground motions which cause a target bridge structure to mainly excite in its first two modes.

5. Conclusions



(a) (b)

Figure 7: The resultant shear force of SC_1 at base of P4: (a) the CMSM for the longitudinal direction (z-direction); and (b) the CMSM for the transverse direction (x-direction).

Table 4: Relative error for the maximum displacement between solid element model and CMSM: (a) the CMSM for the longitudinal direction; and (b) the CMSM for the transverse direction.

(a)

Case	Location	Error / (%)
SC_1	M_1	1.027
SC_2	M_1	1.145
SC_3	M_1	1.403
CC_1	M_1	2.230
CC_2	M_1	1.313
CC_3	M_1	1.953

(b)

Case	Location	Error / (%)
SC_1	M_1	1.201
	M_2	1.206
SC_2	M_1	1.025
	M_2	1.024
SC_3	M_1	1.995
	M_2	1.992
CC_1	M_1	2.195
	M_2	2.192
CC_2	M_1	2.852
	M_2	2.799
CC_3	M_1	2.804
	M_2	2.710

Table 5: Relative error for the maximum resulting shear force at base of P4 between solid element model and CMSM: (a) the CMSM for the longitudinal direction; and (b) the CMSM for the transverse direction.

(a)

Case	Error / (%)
SC_1	2.254
SC_2	2.547
SC_3	3.210
CC_1	3.312
CC_2	3.524
CC_3	3.425

(b)

Case	Error / (%)
SC_1	2.985
SC_2	2.958
SC_3	3.058
CC_1	3.124
CC_2	3.460
CC_3	3.451

In this paper, we propose a consistent mass spring model (CMSM) for fundamental seismic response analysis of a bridge structure. While CMSM contains additional springs in comparison to conventional mass spring models, the CMSM shares fundamental dynamic characteristics and is able to yield structure seismic responses which agree well with those of original models (solid element model). Six different bridge structures are tested successfully in this study. In particular, the resultant force at a specific cross section can be estimated accurately for certain class of ground motions which cause a target bridge structure to mainly excite in its first two modes.

There is a possibility of constructing a more accurate CMSM, by extending the number of mass points. Also, there is a possibility of extending a CMSM to non-linear structure. At least, it is straightforward to apply the meta-modeling to incremental responses of a non-linear elasto-plastic structure.

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