Deck stiffness selection in the design of long span cable stayed bridges

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Abstract

The major objective of the study is to find a useful results on the deck stiffness selection in long span cable stayed bridges at the conceptual stage of design. Decision making involving deck stiffness selection and the choice of the number and spacing of cables to meet deflection criteria is complex and the engineer needs guidance on making early design decisions intuitively. With several assumptions, a model with pinned at one end and clamped at the other end with odd number of cables equally spaced in fan pattern originating from the clamped end was analyzed using stiffness matrix method. At the end of the stiffness matrix analysis, design guidance graphs for (deflection/span length) with deck stiffness was obtained for different number of cables for the worst case of loading for the model with uniformly distributed load throughout the bridge deck and an abnormal type of point loading in the mid span. The deflection limitations for highway bridges in general and cable suspended bridge were included in the graph to obtain the minimum deck stiffness. These, graphs achieved finally from the analysis are worthwhile in the first stage of providing design guidance at the conceptual design in the deck stiffness selection of long span cable stayed bridges.

Keywords: deck stiffness, long span, cable stayed bridges, conceptual stage, deflection

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1. Introduction

Looking at the literature survey of the history of cable stayed bridges, it can be questioned how deck stiffness selection is made for long span cable stayed bridges with the choice of the number of cables. Interesting information on existing live load deflection limitations achieved from the research on various codes of practice and other sources can aid in solving this problem. However, in the past research publications it can be seen that there is an inadequacy issue of conceptual design methods on deck stiffness, cable spacing and span length with deflection of cable stayed bridges. Hence, there is a need and importance in obtaining design guidance on deck stiffness selection in the long span cable stayed bridges at the conceptual design stage.

2. Methodology and analysis

The deck stiffness guidance factors on a real cable stayed bridge, depends as illustrated in *Fig 2.1*.

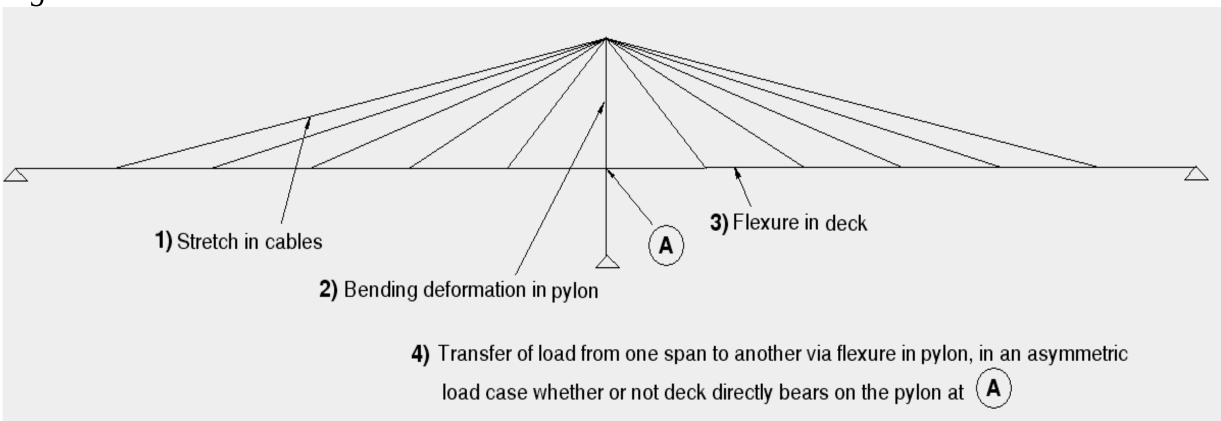

Fig 2.1 The deck stiffness guidance factors

As there are too many factors which make the analysis complex, some assumptions were made to obtain a simpler model. The pylon was considered to be having infinite stiffness to avoid the bending deformation. To avoid the transfer of load from one span to another via flexure in pylon in an asymmetric load case, a model with one end pinned and the other end clamped was chosen. So, for the purpose of analysis, only the stretch in cables and the flexure in deck were taken into account.

If a cable stayed bridge is taken into account, an idealized model with an approximation can be made to the cable stays for stiffeners. So, for the problem in hand, it can be quite easily analyzed using stiffness matrix method.

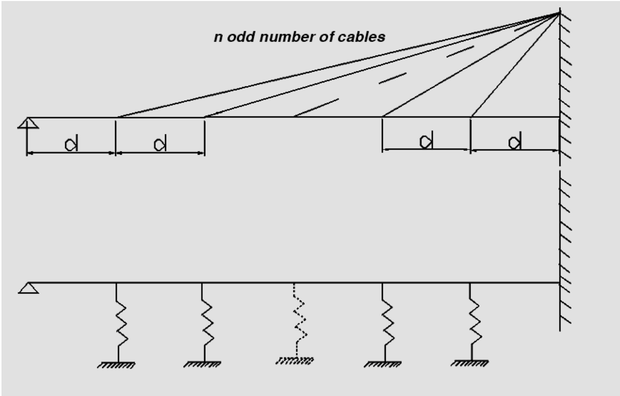

Fig 2.2 Approximation for the idealized model

Still, as there are so many variables which restrict for the purpose of getting useful results on the deck stiffness selection from the analysis, more assumptions were made. The following are the assumptions made for the purpose of analysis.

2.1 Assumptions

1. One end of the bridge deck is pinned and the other end with the pylon is clamped.
2. The pylon at the clamped end of the bridge has infinite stiffness.
3.The stiffness used in the calculations for all cables is the stiffness of the mid cable.
4. The cables are equally spaced throughout the bridge deck.
5. The axial compression at each position of the cable is the axial compression in the mid cable position.
6. All the cables are designed for the 0.2% proof stress of the minimum tensile strength. [1180 M Pa (Gimsing 1997, p. 109)]
7. The bridge has two notional lanes and the cables are originating from the pylon in a fan pattern on the both sides of the road.

In addition, the following other considerations were taken into account:

The total span length of the bridge = 180m
The height of the pylon = 50m
The number of cables (odd numbers) used during the analysis was 3, 7, 17 and 43 with corresponding cable spacing of 45m, 22.5m, 10m and 4.1m respectively.

2.2 The stiffness matrix analysis for the simplified structure

For the simplified structure, with **n** odd number of cables, the joint rotations and displacements were found to be as in *Fig 2.3*.

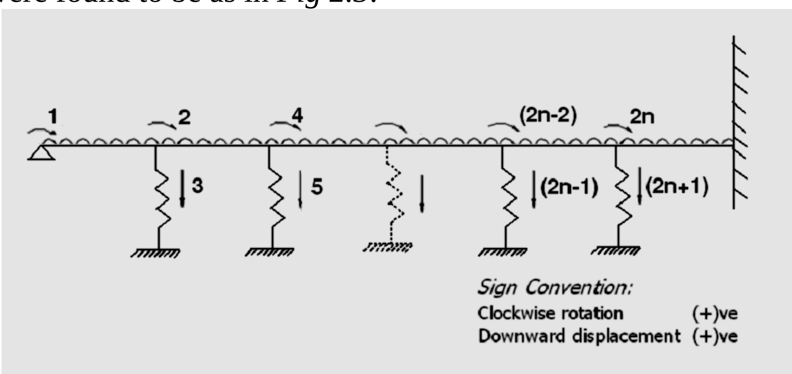

Fig 2.3 Joint rotations and displacement along the bridge deck

Let,

Number of cables = n

Cable spacing = d

The stiffness of the cables = s

Deck stiffness = EI where, E – Young's modulus of the deck

I – Second moment of area of the section

θ_i – Rotation at each node (i = 1, 2, 4, 6,……………,2 n)

δ_i – Vertical displacement at each node [i = 3, 5, 7,….., (2n+1)]

M_i – Moment at each node (i = 1, 2, 4, 6,…………,2n)

F_i – Force at each node [i = 3, 5, 7,…………….., (2n+1)]

M_1	A	(4/d)	(2/d)	$(-6/d^2)$	0	0	0	0	0	0	0	0	0	0	0	0	θ_1	
M_2		(2/d)	(8/d)	0	(2/d)	$(-6/d^2)$	0	0	0	0	0	0	0	0	0	0	θ_2	
F_3		$(-6/d^2)$	0	$(24/d^3) + (s/EI)$	$(6/d^2)$	$(-12/d^3)$	0	0	0	0	0	0	0	0	0	0	δ_3	
M_4		0	(2/d)	$(6/d^2)$	(8/d)	0	(2/d)	$(-6/d^2)$	0	0	0	0	0	0	0	0	θ_4	
F_5		0	$(-6/d^2)$	$(-12/d^3)$	0	$(24/d^3) + (s/EI)$	$(6/d^2)$	$(-12/d^3)$	0	0	0	0	0	0	0	0	δ_5	
-		0	0	0	-	-	-	-	-	-	0	0	0	0	0	0	-	
-	= E I	0	0	0	-	-	-	-	-	-	0	0	0	0	0	0	-	
-		0	0	0	0	0 B	-	-	-	-	-	-	0	0	0	0	-	→ Equation (A)
-		0	0	0	0	0	-	-	-	-	-	-	0	0	0	0	-	
-		0	0	0	0	0	0	0	-	-	-	-	-	-	0	0	-	
-		0	0	0	0	0	0	0	-	-	-	-	-	-	0	0	-	
-		0	0	0	0	0	0	0	0	0	(2/d)	$(6/d^2)$	(8/d)	0	(2/d)	$(-6/d^2)$	-	
$F_{(2n-1)}$		0	0	0	0	0	0	0	0	0	$(-6/d^2)$	$(-12/d^3)$	0	$(24/d^3) + (s/EI)$	$(6/d^2)$	$(-12/d^3)$	$\delta_{(2n-1)}$	
M_{2n}		0	0	0	0	0	0	0	0	0	0	0	(2/d)	$(6/d^2)$	(8/d)	0	θ_{2n}	
$F_{(2n+1)}$		0	0	0	0	0	0	0	0	0	0	0	$(-6/d^2)$	$(-12/d^3)$	0	$(24/d^3) + (s/EI)$	$\delta_{(2n+1)}$	
																C		

In the above stiffness matrix the only unknown is the stiffness of the cable.

Let the uniformly distributed load throughout the bridge deck be ω. Then the joint load vector which is the left hand side of the equation (A) can be found as follows:

M_1		$\omega d^2/12$	
M_2		0	
F_3		ωd	
M_4		0	
F_5		ωd	
-		-	
-	=	-	→ Equation (B)
-		-	
-		-	
-		-	
-		-	
-		-	
$F_{(2n-1)}$		ωd	
M_{2n}		0	
$F_{(2n+1)}$		ωd	

If the Equation (A) can be written in the format of **Z = XY,** as it can be seen from the equation **(A)** & Equation **(B)**, the unknown **Y** can be found in the following manner:

Matrix Y= (Inverse of matrix X) × (Matrix Z)

Hence, from the solution for matrix **Y** the deflection throughout the bridge can be quite easily found by the third component from the top and every other component from the rest.

$$Y = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \delta_3 \\ \theta_4 \\ \delta_5 \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ \delta_{(2n-1)} \\ \theta_{2n} \\ \delta_{(2n+1)} \end{pmatrix}$$

2.3 Programming the stiffness matrix and the joint load vector for *n* odd number of cables

Programming was done using the uniform pattern of the matrices using the programming language ***Microsoft Visual Basic 6.0*** to find the stiffness matrix & joint load vector, and to find the inverse & multiplication of the matrix, ***MATLAB 6.1*** was used for a random number of cable.

Finding a way to change the stiffness of the bridge deck

The major concern of the study is on steel bridges. So a value of 210 G Pa was used during the calculations. Moreover, for the purpose of the cross-section an I-type of cross-section was used. For the I-type of cross-section, the web and flange thickness was kept to the same value of 0.02m. To find the second moment of area, only the flange area needs to be known. So, for the purpose of finding the flange area, the section of the bridge deck in between two cables, the cable spacing, was designed for strength. In the design for the strength calculations of the section, the axial compression acting on the section and the failure due bending moment in the section was taken into account.

2.4 Uniformly distributed load (HA loading)

The normal live loading of the bridge deck, the traffic loading which can be also mentioned as HA loading plays a major role in the deck stiffness selection of the cable stayed bridges. As the worst case of loading will be uniformly distributed load throughout the deck, it was used during the analysis.

Hence, the deflection values along the bridge deck were sorted out from the results which were obtained using the above mentioned programming. Then graphs were plotted for deflection with span length. Then the same procedure was repeated for different depth of the deck. From the graphs of deflection with span length, the maximum deflection values were found. Then the values of deflection/span length were plotted with respective deck stiffness and the results were compared with the deflection limit found for highway bridges in general and the deflection limit for cable suspended bridges in relation to the number of cables.

2.5 The abnormal type of point loading (HB loading)

The other type of loading which also affects the bridge deck stiffness selection is the abnormal loading due to a point load. So, for the purpose of analysis, the worst case of loading was positioned in the mid span of the bridge. For ease of analysis, only odd number of cables was analysed, so that the mid position will coincide with the mid position of the number of cables. An abnormal loading of 45 units as from BS 5400: Part 2:1978 which led to an 1800kN of loading was considered. So, the structure for the point load case can be simplified as in *Fig 2.4*.

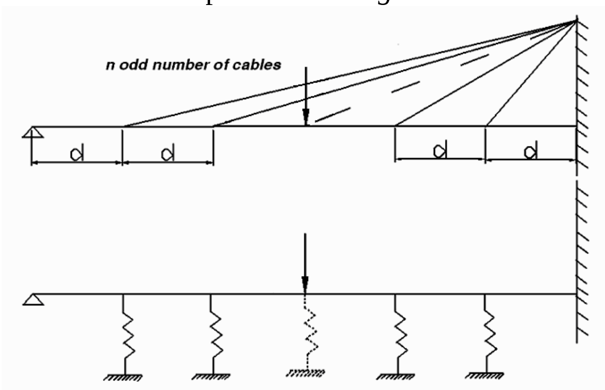

Fig 2.4 Simplified model for the HB type of load case

2.6 The stiffness matrix analysis of the abnormal type of point loading for the simplified model

For the simplified structure, with **n** odd number of cables, the joint rotations and displacements were found to be as in *Fig 2.5*.

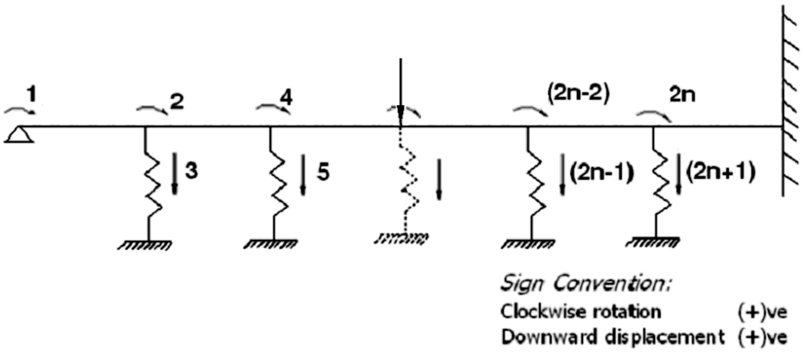

Fig 2.5 Joint rotations and displacement along the bridge deck for the HB type of load case

The similar analysis mentioned for uniformly distributed load can be done for the point load case except for the joint load vector. Let the point load be **P**.

Then, the joint load vector for the point load case in *Fig 2.5* can be as follows:

$$\begin{pmatrix} M_1 \\ M_2 \\ F_3 \\ - \\ - \\ - \\ - \\ F_{n+2} \\ - \\ - \\ - \\ - \\ F_{(2n-1)} \\ M_{2n} \\ F_{(2n+1)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ - \\ - \\ P \\ - \\ - \\ - \\ - \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \text{Equation (B')}$$

The rest will be as same as the uniformly distributed load. The programming was done similarly and graphs were plotted for deflection with span length of the bridge. Then the same procedure was repeated for different depth of the deck. From the graphs of deflection with span length the wave amplitude due to the deflection for the corresponding wave length values were found. Then the (wave amplitude/wave length) values were plotted with respective deck stiffness and the results were compared with the deflection limit found for highway bridges in general and deflection limit for cable suspended bridges in relation to the number of cables.

3. Results and discussion

3.1 Uniformly distributed type of loading throughout the bridge deck

The graph for different number of cables was plotted for deflection Vs span length. The results for this case were found to be as follows in *Fig 3.1*. The results reveal that, except for 3 cables, from 0.8m to 1.5m of the bridge deck depth the deflection have peak values closer to the ends of the span length. Then it decreases and remains constant in a lesser value throughout the mid span. This peak value reduces and after 1.5m deck depth, the deflected shape changes into a parabolic curve with a peak value closer to one third of the span length. This sort of change in deflected shape happens other than for 3 number of cables. The deflection for 43 cables is larger than the other 3, 7 & 17 number of cables for all types of deck depths.

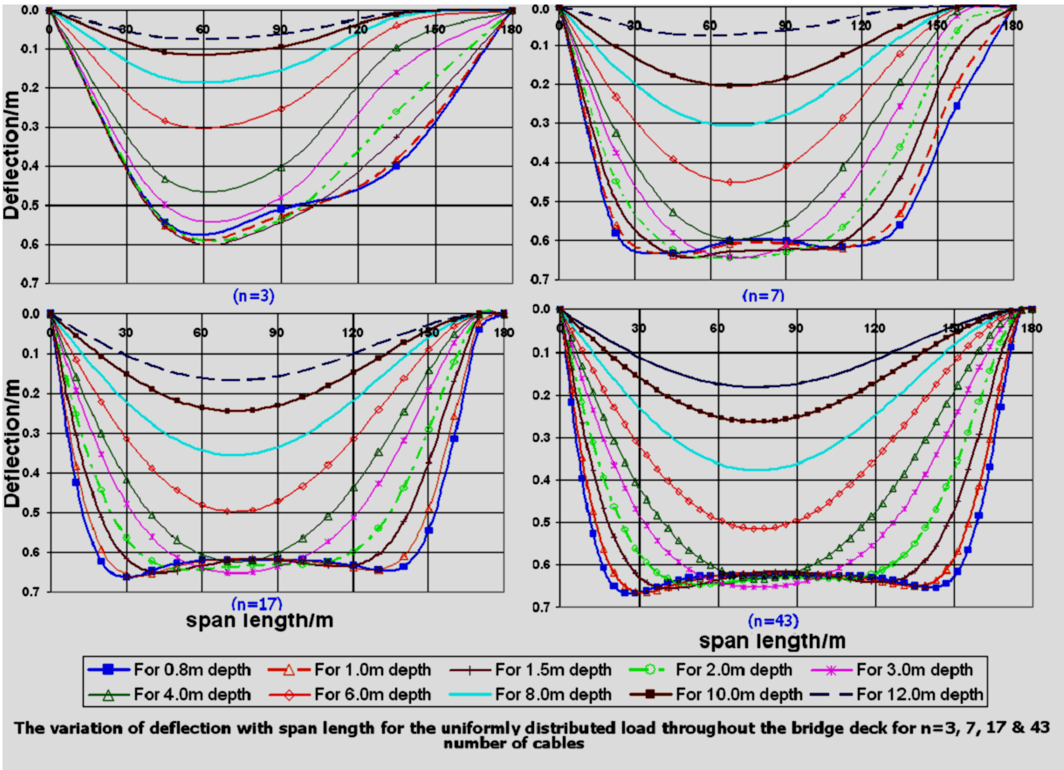

Fig 3.1 Comparison of the variation of deflection with span length for different number of cables for the uniformly distributed load throughout the bridge

3.2 Change in deflection/span length with deck stiffness

From the results of graphs in *Fig 3.1*, the maximum deflection values were obtained to plot a graph for (deflection/span length) with deck stiffness, the EI value. The (deflection/span length) limits for highway bridges in general of the value of (1/800) and cable suspended bridges of the value of (1/300) were also included in the graph. The *Fig 3.2* shows the graph of variation with (deflection/span length) with deck stiffness for different number of cables.

The manner of the graph for all the cables is in a particular nature, with a decrease in (deflection/span length) value for the increase in deck stiffness.

Apparently, the (deflection/span length) values for 43 cables are larger compared with other curves for 3, 7 and 17 number of cables irrespective of the values for deck stiffness. In addition, the (deflection/span length) values for 3 cables seem to be having smaller values compared with all the other number of cables irrespective of the values for deck stiffness.

For 3 cables the curve intersects the deflection limit in a point of 150 GPa m^4 of deck stiffness. For 7 cables, the curve intersects at 320 GPa m^4 of deck stiffness, for 17 cables at 420 GPa m^4 and for 43 cables at 460 GPa m^4. The curves intersect for 17 and 43 cables in the range of (400-500) GPa m^4. In addition, the curves for 7, 17 and 43 number of cables, intersect the deflection limit (1/300) for cable suspended bridges around the value of 30 GPa m^4.

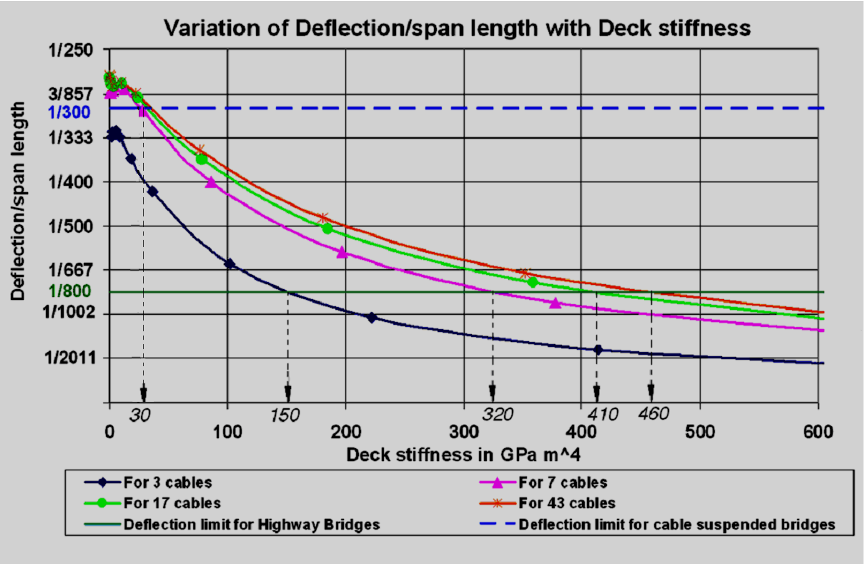

Fig 3.2 Variation of (deflection/span length) with deck stiffness for different number of cables for the uniformly distributed type of load throughout the bridge deck

3.3 Abnormal type of Point loading in the mid span of the bridge

The graphs for different number of cables were plotted for deflection Vs span length for the abnormal type of point loading in the mid span of the bridge. The results for this case were found to be as follows in *Fig 3.3*. When viewing the graphs in *Fig 3.3*, irrespective of the number of cables, the nature of the deflection tends to form a wave form having a larger peak value downwards in the mid span and a peak which is relatively very small compared to the previous one which occurs closer to the mid span in the upward direction. This type of wave form gradually decreases its peak values of the deflection due to point load with the increase in the deck depth. In addition, it tends to disappear as the deck depth starts with a value of 3.0m. Apparently, the 43 cables have larger peak deflection values compared with the other 3, 7 and 17 number of cables. In addition, 3 cables have less deflection values compared to the rest of the number of cables.

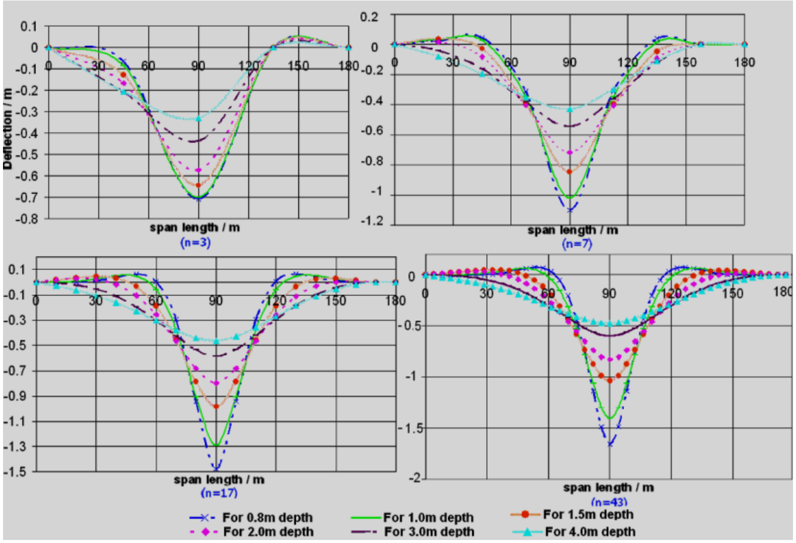

Fig 3.3 Variation of deflection with span length for different number of cables for the abnormal type of loading

3.4 Change in deflection/span length with deck stiffness

With the results of graphs from *Fig 4.3*, the wave amplitude and the distance between the two peak values were found. Then, this wave amplitude was divided by its relevant wave length, the distance along the span length between the peak values. Consequently, graphs were plotted for the (wave amplitude/wave length) with the deck stiffness for each number of cables. The (deflection/span length) limits for highway bridges in general of the value of (1/800) and cable suspended bridges of the value of (1/300) were as well included in the graph.

When viewing the graphs in *Fig 3.4*, except for 3 cables, for the other: 7, 17 and 43 number of cables; the variation in the nature of the graphs are very small. The curves are particular in nature with a decrease in (wave amplitude/wave length) with the increase in deck stiffness. In addition, they intersect the deflection limit for cable suspended bridge in the value of (1/300) in 9 GPa m^2 of deck stiffness value for 3 cables and around 13 GPa m^2 value for all the other cables.

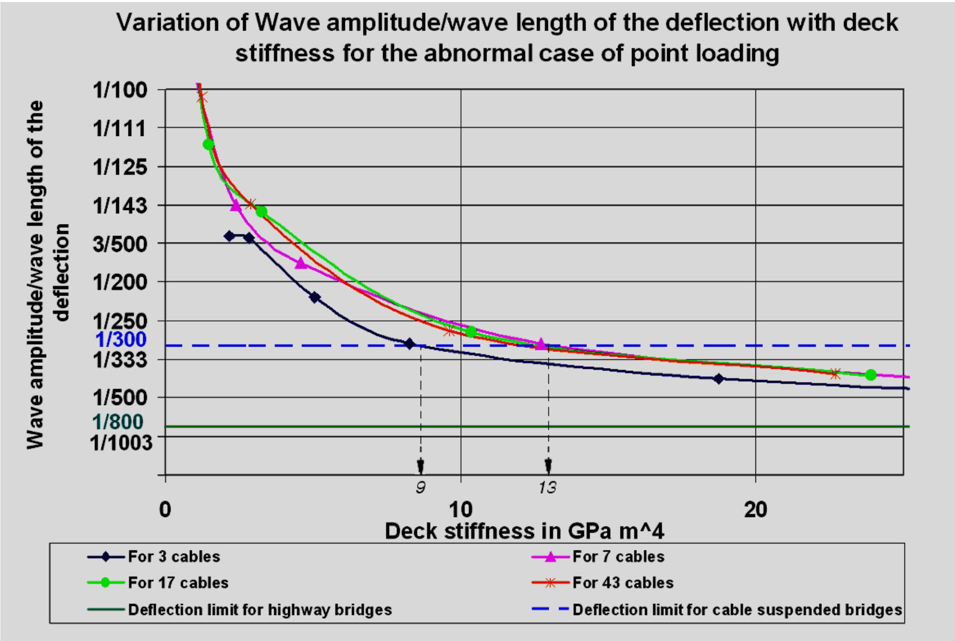

Fig 3.4 Variation of (wave amplitude/wave length) of the deflection with deck stiffness for the abnormal type of load case

4. Discussion

The reader must keep in mind that as there were several assumptions made during the analysis, it must have affected the nature of the results from the reality. Even though, this type of results achieved is useful in the deck stiffness selection process at the conceptual design stage.

One of the assumptions, the stiffness of all the cables was assumed to be the stiffness of the mid cable. But in reality, all the cables will have different stiffness.

When finding the stiffness of the cable, only the stretch in the cable was taken into account. But, there is a contraction due to the axial compression in the deck also involves. As the bridge deck is very large in size and infinite stiffness in comparison with the stretch in cables, the value of contraction will be very small.

Another, assumption assumed was the axial compression in the mid cable position is in all the cable positions. But in reality, the axial compression differs along the bridge deck.

In reality, a different section sizing will be chosen along bridge deck. Furthermore, the bending moment in the bridge deck was considered as $F\ell^2/8$ considering pinned at both ends. But, it can't be the extreme case of bending moment due to live load. In reality, it can be a different value. A check was done to find the deviation of the assumed bending moment from the calculated moment due to the results obtained from the solution of stiffness matrix analysis. For the purpose of the calculations, 43 number of cables with a cable spacing of 4.1m was taken into considerations. The calculated bending moment from the results obtained from the assumed value, is fairly a large value in comparison with the assumed value.

Moreover, a very slender I-beam section with 0.02m thickness for the flange and web was chosen for the calculation of bending moment in the programming to change the depth. In real situation, a varying thickness of flange and web with lesser value of depth will be chosen to avoid lateral torsional buckling and shear links will be provided to avoid the web buckling failure.

For the point load type of loading case in the mid span deck depth of the range of (0.8 - 4.0) m was considered during the analysis to obtain the results, whereas for the uniformly distributed type of load case throughout the bridge deck, (0.8-12) m was taken into account. The reason behind this was, as the final graphs were plotted for (wave amplitude/wave length), the nature of the wave forms in the graphs in *Fig 3.3*, tends to disappear nearly as it reaches 4.0m of deck depth. So the deck depth wasn't chosen further than that value.

Eventually, the final curves obtained in *Fig 3.4*, for (wave amplitude/wave length) with deck stiffness seem to be not very smooth. When assuming to be a beam on elastic foundation, the position of the stiffeners will be in discrete points rather than in the real case a continuous deck filled with elastic material. In addition, the wave amplitude and wave length were measured manually with the graphs obtained in *Fig 3.3*, which has an inaccuracy problem with the values obtained. Hence, it led to not very smooth curves. Even though, with all these assumptions and inaccuracy problem, the final results achieved is meaningful.

5. Conclusion and future recommendations

5.1 Conclusion

5.1.1 Uniformly distributed type of loading throughout the bridge deck

From the analysis, graphs for deflection along the bridge span were plotted for different number of cables. For 3 cables the curve intersects the deflection limit in a point of 150 GPa m^4 of deck stiffness. The curves intersect for 7, 17 and 43 cables in the range of (300-500) GPa m^4. In addition, the curves for 7, 17 and 43 number of cables, intersect the deflection limit (1/300) for cable suspended bridges around the value of 30 GPa m^4.

5.1.2 Abnormal type of Point loading in the mid span of the bridge

The graphs were plotted for the (wave amplitude/wave length) of deflection Vs deck stiffness for 7, 17 & 43. Except for 3 cables, for the other: 7, 17 and 43 number of cables; the variation in the nature of the graphs are very small. They intersect the deflection limit for cable suspended bridge in the value of (1/300) in 9 GPa m^2 of deck stiffness value for 3 cables, and around 13 GPa m^2 value for all the other cables.

Even though, with all the assumptions and inaccuracy problem during the process of analysis and obtaining the final results, the achievement for the variation with (deflection/span length) and deck stiffness for different number of cables for the worst case of loadings is meaningful and useful. Moreover, it is a first stage of providing design guidance in the conceptual design stage of long span cable stayed bridges.

5.2 Future Recommendations

As deck stiffness selection in long span bridges is very important, the same type of analysis can be done with the elimination of some of the assumptions used during the analysis. They can be such as, variable stiffness of the cables and flexible pylon. The same type of analysis can also be done for concrete bridges.

References

1. AMMANN, O.H., (1933). "George Washington Bridge: General conception and Development of design".,*American society of Civil Engineers*, Vol.97, *Transaction paper No. 1818*

2. BANGASH, M.Y.H., (1999). "Prototype bridge structures: analysis and design", *London: Thomas Telford Limited*.

3. BELL, A.J., (1968). "An investigation into the spacing of hangers on suspension bridges". *Thesis (PhD). University of Manchester*

4. CICALA.P., (1991). "On the history of suspension bridge theory", *The Institution of Civil Engineers*, pp 345-351

5. CROLL, J.G.A., (1997). "Thoughts on the structural efficiency of cable-stayed and catenary suspension bridges"., The Structural Engineer, Vol.75, No. 10, 20 May, 173-175

7. DALTON, H.C., (1997). Correspondance "Thoughts on the structural efficiency of cable-stayed and catenary suspension bridges"., *The Structural Engineer*, Vol. 75/No 19, 7 October, 345-347.

8. ENDE, M.A., (1905). "Long span bridges", *Professional papers of the Corps of Royal Engineers (Fourth series)*, Vol I –Paper 2

9. GERARD, F. (1980). "Long-span Bridges", *O.H.Ammann Centennial Conf., Annals of the New York Academy of Sciences* 352, pp 87-102

11. GIMSING, N.J., (1991). "Structural systems for cable suspended bridges", *CISM Courses and Lectures No.316 International centre for Mechanical sciences*, December, 69-88

12. GIMSING, N.J., (1997). "Cable Supported Bridges Concept and Design", 2nd edn. *West Sussex: John Wiley & Sons.*