Inverse Reliability Analysis Using High Dimensional Model Representation

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Abstract

Reliability analysis is one of the major concerns at the design stage since the occurrence of failures in engineering systems may lead to catastrophic consequences. Therefore, the expectation of higher reliability and lower environmental impact has become imperative. Hence the inverse reliability problem arises when one is seeking to determine the unknown design parameters such that prescribed reliability indices are attained. The inverse reliability problems with implicit response functions require the evaluation of the derivatives of the response functions with respect to the random variables. When these functions are implicit functions of the random variables, derivatives of these response functions are not readily available. Moreover in many engineering systems, due to unavailability of sufficient statistical information, some uncertain variables cannot be modelled as random variables. In this paper High Dimensional Model Representation (HDMR) based inverse reliability analysis method is presented for the determination of the design parameters in the presence of mixed uncertain variables. The method involves HDMR approximation of the limit state function, transformation technique to obtain the contribution of the fuzzy variables to the convolution integral, and fast Fourier transform techniques to evaluate the convolution integral for solving the inverse reliability problem. The accuracy and efficiency of the proposed method is demonstrated through two numerical examples.

Keywords: Inverse reliability analysis, High Dimensional Model Representation, Random variables, Fuzzy variables, Convolution integral

1. Introduction

The solution procedure for inverse reliability problems is required to determine the unknown design parameters such that prescribed reliability indices are attained. One way to solve the inverse reliability problem is through trial and error procedure, using a forward reliability method like first-order reliability method (FORM) and varying the design parameters until the achieved reliability index matches the required target (Li and Foschi, 1998). However, the trial and error procedures are inefficient and involve difficulties resulting from repetitive forward reliability analysis. As a result there is considerable interest in developing an efficient and more direct approach to determine the design parameters for a specified target reliability level. An inverse first-order reliability method (inverse FORM) was developed for the estimation of design loads associated with specified target reliability levels in offshore structures (Winterstein *et al*. 1993), later it was extended to general limit state functions (Kiureghian et al., 1994). To overcome the drawbacks of the inverse FORM, artificial neural network (ANN)-based inverse FORM (Cheng *et al*. 2007) as well as polynomial-based response surface method (Cheng and Li, 2009) were developed. The most probable point based dimension reduction method was developed for reliability-based design optimization of nonlinear and multi-dimensional systems (Lee *et al*. 2008)

Traditionally, inverse reliability methods require complete statistical information of uncertainties. These uncertainties are treated stochastically and assumed to follow certain probability distributions. However, in many practical engineering applications, the distributions of some random variables may not be precisely known or uncertainties may not be appropriately represented with probability distributions. Consequently the non-probabilistic approach (Ben-Haim, 1995) has been rapid developed for describing uncertainty with incomplete statistical information by a fuzzy set or a convex set. However all the methods discussed above consider either random variables or fuzzy input, but do not accommodate a combination of both types of variables. In the design problem with both statistical random variables and fuzzy variables, if the random variables are converted into fuzzy variables by generating membership functions, the method may yield a design that is too conservative because treating the random variables as fuzzy variables loses accuracy of the uncertainties. On the other hand, treating fuzzy variables as random variables by adopting approximate probability distributions may lead to an unreliable optimum design. Therefore, in this paper a novel solution procedure for inverse reliability problems with implicit response functions without requiring the derivatives of the response functions with respect to the uncertain variables, is proposed to determine the unknown design parameters such that prescribed reliability indices are attained in the presence of mixed uncertain (both random and fuzzy) variables.

2. High Dimensional Model Representation

High Dimensional Model Representation (HDMR) is a general set of quantitative model assessment and analysis tools for capturing the high-dimensional relationships between sets of input and output model variables. It is a very efficient formulation of the system response, if higher order variable correlations are weak, allowing the physical model to be captured by the first few lower order terms. Practically for most well-defined physical systems, only relatively low order correlations of the input variables are expected to have a significant effect on the overall response. HDMR expansion utilizes this property to present an accurate hierarchical representation of the physical system (Rabitz and Alis, 1999; Rao and Chowdhury, 2008). Let the N−dimensional vector $\boldsymbol{x}=\{x_1,x_2,\ldots,x_N\}$ represent the input variables of the model under consideration, and the response function as $g(\boldsymbol{x})$. Since the influence of the input variables on the response function can be independent and/or cooperative, HDMR expresses the response $g(\boldsymbol{x})$ as a hierarchical correlated function expansion in terms of the input variables as

$$\begin{aligned} g(\boldsymbol{x}) = g_0 + \sum_{i=1}^{N} g_i(x_i) + \sum_{1\le i_1<i_2\le N} g_{i_1 i_2}(x_{i_1},x_{i_2}) + \ldots \\ + \sum_{1\le i_1<\ldots<i_l\le N} g_{i_1 i_2 \ldots i_l}(x_{i_1},x_{i_2},\ldots,x_{i_l}) + \ldots + g_{12\ldots N}(x_1,x_2,\ldots,x_N) \end{aligned}, \tag{1}$$

where g_0 is a constant term representing the zeroth-order component function or the mean response of $g(\boldsymbol{x})$. The function $g_i(x_i)$ is a first-order term expressing the effect of variable x_i acting alone, although generally nonlinearly, upon the output $g(\boldsymbol{x})$. The function $g_{i_1 i_2}(x_{i_1},x_{i_2})$ is a second-order term which describes the cooperative effects of the variables x_{i_1} and x_{i_2} upon the output $g(\boldsymbol{x})$. The higher order terms give the cooperative effects of increasing numbers of input variables acting together to influence the output $g(\boldsymbol{x})$. The last term $g_{12,\cdots,N}(x_1,x_2,\cdots,x_N)$ contains any residual dependence of all the input variables locked together in a cooperative way to influence the output $g(\boldsymbol{x})$. Once all the relevant component functions in Eq. (1) are determined and suitably represented, then the component functions constitute HDMR, thereby replacing the original computationally expensive method of calculating $g(\boldsymbol{x})$ by the computationally efficient model. The expansion functions are determined by evaluating the input-output responses of the system relative to the defined reference point $\boldsymbol{c}$ along associated lines, surfaces, subvolumes, etc. in the input variable space. This process reduces to the following relationship for the component functions in Eq. (1),

$$g_0 = g(\boldsymbol{c}), \tag{2}$$

$$g_i(x_i) = g(x_i,\boldsymbol{c}^i) - g_0, \tag{3}$$

$$g_{i_1 i_2}(x_{i_1},x_{i_2}) = g(x_{i_1},x_{i_2},\boldsymbol{c}^{i_1 i_2}) - g_{i_1}(x_{i_1}) - g_{i_2}(x_{i_2}) - g_0, \tag{4}$$

where the notation $g(x_i,\boldsymbol{c}^i) = g(c_1,c_2,\ldots,c_{i-1},x_i,c_{i+1},\ldots,c_N)$ denotes that all the input variables are at their reference point values except x_i. The g_0 term is the output response of the system evaluated at the reference point $\boldsymbol{c}$. The higher order terms are evaluated as cuts in the input variable space through the reference point. Therefore, each first-order term $g_i(x_i)$ is evaluated along its variable axis through the reference point. Each second-order term $g_{i_1 i_2}(x_{i_1},x_{i_2})$ is evaluated in a plane defined by the binary set of input variables x_{i_1} and x_{i_2} through the reference point, etc. Considering terms up to first-order in Eq. (1) yields,

$$g(\boldsymbol{x}) = g_0 + \sum_{i=1}^{N} g_i(x_i) + \mathrm{R}_2. \tag{5}$$

Substituting Eq. (2) and Eq. (3) into Eq. (5) leads to

$$g(\boldsymbol{x}) = \sum_{i=1}^{N} g(c_1,\ldots,c_{i-1},x_i,c_{i+1},\ldots,c_N) - (N-1)g(\boldsymbol{c}) + \mathrm{R}_2 . \tag{6}$$

Now consider first-order approximation of $g(\boldsymbol{x})$, denoted by

$$\tilde{g}(\boldsymbol{x}) \equiv g(x_1,x_2,\ldots,x_N) = \sum_{i=1}^{N} g(c_1,\ldots,c_{i-1},x_i,c_{i+1},\ldots,c_N) - (N-1)g(\boldsymbol{c}). \tag{7}$$

Comparison of Eq. (6) and Eq. (7) indicates that the first-order approximation leads to the residual error $g(\boldsymbol{x}) - \tilde{g}(\boldsymbol{x}) = \mathrm{R}_2$, which includes contributions from terms of two and higher order component functions. The notion of 0th, 1st, etc. in HDMR expansion should not be confused with the terminology used either in the Taylor series or in the conventional least-squares based regression model. It can be shown that, the first order component function $g_i(x_i)$ is the sum of all the Taylor series terms which contain and only contain variable x_i. Hence first-order HDMR approximations should not be viewed as first-order Taylor series expansions nor do they limit the nonlinearity of $g(\boldsymbol{x})$. Furthermore, the approximations contain contributions from all input variables. Thus, the infinite number of terms in the Taylor series is partitioned into finite different groups and each group corresponds to one cut-HDMR component function. Therefore, any truncated cut-HDMR expansion provides a better approximation and convergent solution of $g(\boldsymbol{x})$ than any truncated Taylor series because the latter only contains a finite number of terms of Taylor series. Furthermore, the coefficients associated with higher dimensional terms are usually much smaller than that with one-dimensional terms. As such, the impact of higher dimensional terms on the function is less, and therefore, can be neglected. Compared with the FORM and SORM which retain only linear and quadratic terms, respectively, first-order HDMR approximation $\tilde{g}(\boldsymbol{x})$ provides more accurate representation of the original implicit limit state function $g(\boldsymbol{x})$.

3. Inverse Structural Reliability Analysis Using HDMR and FFT

The objective of the inverse reliability analysis using HDMR and FFT is to find a new MPP, denoted by $\boldsymbol{x}^*_{\mathrm{HDMR}}$, which will be then used in the subsequent iteration of analysis. The proposed computational procedure involves the following three steps: estimation of failure probability in presence of mixed uncertain variables, reliability index update, and MPP update.

3.1 Estimation of Failure Probability in Presence of Mixed Uncertain Variables

Let the N-dimensional input variables vector $\boldsymbol{x} = \{x_1,x_2,\ldots,x_N\}$, which comprises of r number of random variables and f number of fuzzy variables be divided as, $\boldsymbol{x} = \{x_1,x_2,\ldots,x_r,x_{r+1},x_{r+2},\ldots,x_{r+f}\}$ where the subvectors $\{x_1,x_2,\ldots,x_r\}$ and $\{x_{r+1},x_{r+2},\ldots,x_{r+f}\}$ respectively group the random variables and the fuzzy variables, with $N = r + f$. Then the first-order approximation of $\tilde{g}(\boldsymbol{x})$ in Eq. (7) can be divided into three parts, the first part with only the random variables, the second part with only the fuzzy variables and the third part is a constant which is the output response of the system evaluated at the reference point $\boldsymbol{c}$, as follows

$$\tilde{g}(\boldsymbol{x}) = \sum_{i=1}^{r} g\left(x_i, \boldsymbol{c}^i\right) + \sum_{i=r+1}^{N} g\left(x_i, \boldsymbol{c}^i\right) - (N-1) g(\boldsymbol{c}). \tag{8}$$

The joint membership function of the fuzzy variables part is obtained using suitable transformation of the variables $\{x_{r+1}, x_{r+2}, \ldots, x_N\}$ and interval arithmetic algorithm. Using this approach, the minimum and maximum values of the fuzzy variables part are obtained at each α-cut. Using the bounds of the fuzzy variables part at each α-cut along with the constant part and the random variables part in Eq. (8), the joint density functions are obtained by performing the convolution using FFT in the rotated Gaussian space at the MPP, which upon integration yields the bounds of the failure probability.

3.1.1 Transformation of Interval Variables

Optimization techniques are required to obtain the minimum and maximum values of a nonlinear response within the bounds of the interval variables. This procedure is computationally expensive for problems with implicit limit state functions, as optimization requires the function value and gradient information at several points in the iterative process. But, if the function is expressed as a linear combination of interval variables, then the bounds of the response can be expressed as the summation of the bounds of the individual variables. Therefore, fuzzy variables part of the nonlinear limit state function in Eq. (8) is expressed as a linear combination of intervening variables by the use of first-order HDMR approximation in order to apply an interval arithmetic algorithm, as follows

$$\sum_{i=r+1}^{N} g\left(x_i, \boldsymbol{c}^i\right) = z_1 + z_2 + \ldots + z_f, \tag{9}$$

where $z_i = \left(\beta_i x_i + \gamma_i\right)^{\kappa}$ is the relation between the intervening and the original variables with κ being order of approximation taking values $\kappa = 1$ for linear approximation, $\kappa = 2$ for quadratic approximation, $\kappa = 3$ for cubic approximation, and so no. The bounds of the intervening variables can be determined using transformations (Adduri and Penmetsa, 2008). If the membership functions of the intervening variables are available, then at each α-cut, interval arithmetic techniques can be used to estimate the response bounds at that level.

3.1.2 Estimation of Failure Probability using FFT

Concept of FFT can be applied to the problem if the limit state function is in the form of a linear combination of independent variables and when either the marginal density or the characteristic function of each basic random variable is known. In the present study HDMR concepts are used to express the random variables part along with the values of the constant part and the fuzzy variables part at each α-cut as a linear combination of lower order component functions. The steps involved in the proposed method for failure probability estimation as follows:

(i) If $\boldsymbol{u} = \{u_1, u_2, \ldots, u_r\}^T \in \Re^r$ is the standard Gaussian variable, let $\boldsymbol{u}^* = \{u_1^*, u_2^*, \ldots, u_r^*\}^T$ be the MPP or design point, determined by a standard nonlinear constrained optimization. The MPP has a distance β_{HL}, which is commonly referred to as the Hasofer–Lind reliability index. Construct an orthogonal matrix $\boldsymbol{R} \in \Re^{r \times r}$ whose r–th column is $\boldsymbol{\alpha}^* = \boldsymbol{u}^* / \beta_{HL}$, i.e., $\boldsymbol{R} = [\boldsymbol{R}_1 \,|\, \boldsymbol{\alpha}^*]$ where $\boldsymbol{R}_1 \in \Re^{r \times r-1}$ satisfies $\boldsymbol{\alpha}^{*T} \boldsymbol{R}_1 = 0 \in \Re^{1 \times r-1}$. The matrix $\boldsymbol{R}$ can be obtained, for example, by

Gram–Schmidt orthogonalization. For an orthogonal transformation $\boldsymbol{u} = \boldsymbol{R}\boldsymbol{v}$. Let $\boldsymbol{v} = \{v_1, v_2, \ldots, v_r\}^T \in \Re^r$ be the rotated Gaussian space with the associated MPP $\boldsymbol{v}^* = \{v_1^*, v_2^*, \ldots, v_r^*\}^T$. Note that in the rotated Gaussian space the MPP is $\boldsymbol{v}^* = \{0, 0, \ldots, \beta_{HL}\}^T$. The transformed limit state function $g(\boldsymbol{v})$ therefore maps the random variables along with the values of the constant part and the fuzzy variables part at each α-cut, into rotated Gaussian space $\boldsymbol{v}$. First-order HDMR approximation of $g(\boldsymbol{v})$ in rotated Gaussian space $\boldsymbol{v}$ with $\boldsymbol{v}^* = \{v_1^*, v_2^*, \ldots, v_r^*\}^T$ as reference point can be represented as follows:

$$\tilde{g}(\boldsymbol{v}) \equiv g(v_1, v_2, \ldots, v_r) = \sum_{i=1}^{r} g\left(v_1^*, \ldots, v_{i-1}^*, v_i, v_{i+1}^*, \ldots, v_r^*\right) - (r-1)\, g\left(\boldsymbol{v}^*\right). \tag{10}$$

(ii) In addition to the MPP as the chosen reference point, the accuracy of first-order HDMR approximation in Eq. (10) may depend on the orientation of the first $r-1$ axes. In the present work, the orientation is defined by the matrix $\boldsymbol{R}$. In Eq. (10), the terms $g(v_1^*, \ldots, v_{i-1}^*, v_i, v_{i+1}^*, \ldots, v_r^*)$ are the individual component functions and are independent of each other. Eq. (10) can be rewritten as,

$$\tilde{g}(\boldsymbol{v}) = a + \sum_{i=1}^{r} g\left(v_i, \boldsymbol{v}^{*i}\right), \tag{11}$$

where $a = -(r-1)g(\boldsymbol{v}^*)$.

(iii) New intermediate variables are defined as

$$y_i = g\left(v_i, \boldsymbol{v}^{*i}\right). \tag{12}$$

The purpose of these new variables is to transform the approximate function into the following form

$$\tilde{g}(\boldsymbol{v}) = a + y_1 + y_2 + \cdots + y_r. \tag{13}$$

(iv) Due to rotational transformation in $\boldsymbol{v}$-space, component functions y_i in Eq. (13) are expected to be linear or weakly nonlinear function of random variables v_i. In this work both linear and quadratic approximations of y_i are considered. Let $y_i = b_i + c_i\, v_i$ and $y_i = b_i + c_i\, v_i + e_i\, v_i^2$ be the linear and quadratic approximations, where coefficients $b_i \in \Re$, $c_i \in \Re$ and $e_i \in \Re$ (non-zero) are obtained by least-squares approximation from exact or numerically simulated conditional responses $\left\{g\left(v_i^1, \boldsymbol{v}^{*i}\right), g\left(v_i^2, \boldsymbol{v}^{*i}\right), \cdots, g\left(v_i^n, \boldsymbol{v}^{*i}\right)\right\}^T$ at n sample points along the variable axis v_i. Then Eq. (13) results in

$$\tilde{g}(\boldsymbol{v}) \equiv a + y_1 + y_2 + \cdots + y_r = a + \sum_{i=1}^{r} \left(b_i + c_i\, v_i\right), \tag{14}$$

and

$$\tilde{g}(\boldsymbol{v}) \equiv a + y_1 + y_2 + \cdots + y_r = a + \sum_{i=1}^{r} \left(b_i + c_i\, v_i + e_i\, v_i^2\right). \tag{15}$$

The least-squares approximation is chosen over interpolation, because the former minimizes the error when $n > 2$ for linear and $n > 3$ for quadratic approximations.

(v) Since v_i follows standard Gaussian distribution, marginal density of the intermediate variables y_i can be easily obtained by simple transformation (using chain rule).

$$p_{Y_i}(y_i) = p_{V_i}(v_i)\left|\frac{1}{dy_i/dv_i}\right|. \tag{16}$$

(vi) Now the approximation is a linear combination of the intermediate variables y_i. Therefore, the joint density of $\tilde{g}(\boldsymbol{v})$, which is the convolution of the individual marginal density of the intervening variables y_i, can be expressed as follows:

$$p_{\tilde{G}}(\tilde{g}) = p_{Y_1}(y_1) * p_{Y_2}(y_2) * \ldots * p_{Y_r}(y_r), \tag{17}$$

where $p_{\tilde{G}}(\tilde{g})$ represents joint density of the transformed limit state function $\tilde{g}(\boldsymbol{v})$.

(vii) Applying FFT on both sides of Eq. (17), leads to

$$FFT\left[p_{\tilde{G}}(\tilde{g})\right] = FFT\left[p_{Y_1}(y_1)\right]FFT\left[p_{Y_2}(y_2)\right]\ldots FFT\left[p_{Y_r}(y_r)\right]. \tag{18}$$

(viii) By applying inverse FFT on both side of Eq. (18), joint density of the limit state function $\tilde{g}(\boldsymbol{v})$ is obtained.

(ix) The probability of failure is given by the following equation

$$P_F^{\text{HDMR}} = \int_{-\infty}^{0} p_{\tilde{G}}(\tilde{g})\,d\tilde{g}. \tag{19}$$

After computing the probability of failure P_F^{HDMR} using coupled HDMR-FFT technique, the corresponding reliability index β_{HDMR} can be obtained by

$$\beta_{\text{HDMR}} = -\Phi^{-1}\left(P_F^{\text{HDMR}}\right), \tag{20}$$

where $\Phi(\bullet)$ is the cumulative distribution function of a standard Gaussian random variable.

3.2 Reliability Index and MPP Update Procedure

As expected it is very likely that the β_{HDMR} (computed using Eq. (20)) is not the same as the target reliability index $\beta_t = -\Phi^{-1}(P_F^{\text{Tar}})$, and hence, using the difference between these two reliability indices, a recursive formula is obtained as

$$\beta^{(k+1)} \cong \beta^{(k)} - \left(\beta_{\text{HDMR}} - \beta_t\right), \tag{21}$$

where $\beta^{(k)}$ is the reliability index at the current step, with $\beta^{(0)} = \beta_t$ at the initial step.

The updated MPP is approximated as

$$\boldsymbol{u}_{k+1}^* \cong \frac{\beta^{(k+1)}}{\beta^{(k)}}\boldsymbol{u}_k^* \quad \text{or} \quad \boldsymbol{v}_{k+1}^* \cong \frac{\beta^{(k+1)}}{\beta^{(k)}}\boldsymbol{v}_k^*, \tag{22}$$

The updated MPP obtained through Eq. (22) is called the coupled HDMR-FFT based MPP, denoted by $\boldsymbol{u}_{\text{HDMR}}^*$ in $\boldsymbol{U}$-space or $\boldsymbol{x}_{\text{HDMR}}^*$ in $\boldsymbol{X}$-space.

3.3 Detailed Algorithm of Proposed Computational Procedure

The various steps involved in the proposed computational procedure for inverse reliability problems with implicit response functions in the presence of mixed uncertain variables are as follows:

(i) Find MPP in the rotated Gaussian space using a given reliability index $\beta^{(k)}$.

(ii) Calculate the probability of failure P_F^{HDMR} and the corresponding reliability index β_{HDMR} using coupled HDMR-FFT technique.

(iii) Using Eq. (21) and Eq. (22) respectively, update the reliability index from $\beta^{(k)}$ to $\beta^{(k+1)}$ and MPP from $\boldsymbol{u}_k^*$ ($\boldsymbol{v}_k^*$) to $\boldsymbol{u}_{k+1}^*$ ($\boldsymbol{v}_{k+1}^*$).

(iv) Find a new coupled HDMR-FFT based MPP $\boldsymbol{x}_{\text{HDMR}}^*$.

(v) Compare $\boldsymbol{x}_{\text{HDMR}}^*$ and $\boldsymbol{x}^*$.

(vi) Repeat Steps 1–5 until converged.

(vii) Using the minimum and maximum values of the fuzzy variables part at each α-cut, the bounds of the design variables can be obtained by adopting the above procedure.

4. Numerical Examples

To obtain the approximation of the HDMR component functions of fuzzy variables part of the nonlinear limit state function in Eq. (12), n sample points x_{iL}, $x_{iM}-(n-3)(x_{iM}-x_{iL})/(n-1)$, $x_{iM}-(n-5)(x_{iM}-x_{iL})/(n-1)$, …, x_{iM}, …, $x_{iM}+(n-5)(x_{iU}-x_{iM})/(n-1)$, $x_{iM}+(n-3)(x_{iU}-x_{iM})/(n-1)$, x_{iU} are deployed along axis of each of the fuzzy variable x_i having triangular membership function with the triplet number $[x_{iL}, x_{iM}, x_{iU}]$. Similarly to obtain linear/quadratic approximation of the HDMR component functions of random variables part of the nonlinear limit state function in Eq. (12), n sample points $\mu_i-(n-1)\sigma_i/2$, $\mu_i-(n-3)\sigma_i/2$, …, μ_i, …, $\mu_i+(n-3)\sigma_i/2$, $\mu_i+(n-1)\sigma_i/2$ are deployed along axis of each of the random variable x_i with mean μ_i and standard deviation σ_i. If N and n respectively denote the number of uncertain variables, the number of sample points taken along each of the variable axis, then using first-order HDMR approximation the total cost of original function evaluation entails a maximum of $N\times(n-1)+1$ by the proposed method. The efficiency and robustness of the proposed method is expected to increase with increase in the complexity of the structure, number of uncertain variables.

4.1 Hypothetical Limit State Function

This example considers a four dimensional quadratic function of the following form:

$$g(\boldsymbol{x}) = -x_1^2 - x_2^2 - x_3^2 - x_4^2 + 9x_1 + 11x_2 + 11x_3 + 11x_4 - 95.5, \qquad (23)$$

where $x_1 - x_3$ are assumed to be independent normal variables with $N(5, 0.4)$. The variable x_4 is assumed as fuzzy represented by the triplet $[4.6,\ 5.0,\ 5.4]$. The objective is to find the membership functions of $x_1^* - x_3^*$ values, such that the target reliability index $\beta_t = 1.645$ (which corresponds to a failure probability $P_F = 0.05$) is achieved. The presence of mixed uncertain (both random and fuzzy) variables, leads to the membership function of MPP ($x_1^* - x_3^*$) instead of having a unique value at the target reliability index. Figs. 1(a)– 1(c) respectively show the membership functions of $x_1^* - x_3^*$ values at the target reliability index estimated by the proposed method using linear and quadratic approximations. The effect of number of sample points is studied by varying n from 3 to 9. In Figs. 1(a)–1(c) it can be observed that the membership functions of $x_1^* - x_3^*$ values estimated by the proposed method using $n = 7$ and 9 are overlapping each other.

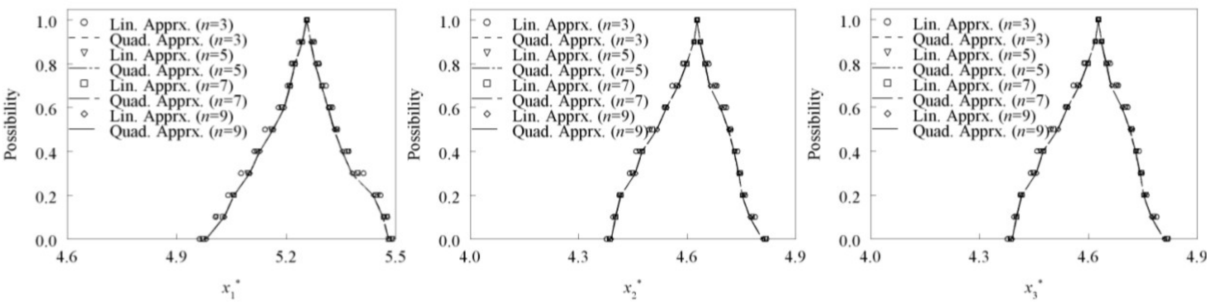

Figure 1: Membership function of design variables (a) x_1^*; (b) x_2^*; and (c) x_3^*

4.2 Single Story Linear Frame Structure

In this example a linear frame structure of one story and one bay as shown in Fig. 2(a) is considered. The cross sectional areas A_1 and A_2 are assumed to be log-normally distributed random variables with mean values of 0.36 and 0.18, and standard deviation values of 0.036 and 0.018 respectively. The horizontal load P is treated as fuzzy with a triplet of [15, 20, 25]. The sectional moments of inertia are expressed as $I_i = \alpha_i A_i^2$ ($i = 1, 2$; $\alpha_1 = 0.08333$, $\alpha_2 = 0.16670$). The Young's modulus E is treated as deterministic. $E = 2.0 \times 10^6$ kN/m^2. In this study, the functional relationship to define the horizontal displacement at the top of the frame is:

$$g(A_1, A_2, P) = \Delta_{\text{lim}} - u_h, \tag{24}$$

where Δ_{lim} is taken as 10 mm. Our interest is to find A_1^* and A_2^*, such that the target reliability index $\beta_t = 2.831$ (which corresponds to a failure probability $P_F = 2.32 \times 10^{-3}$) is achieved.

The implicit limit state function given in Eq. (24) is approximated using first-order HDMR by deploying n sample points along each of the variable axis. The approximated limit state function is divided into two parts, one with only the random variables along with the value of the constant part, and the other with the fuzzy variables. The joint membership function of the fuzzy part of approximated limit state function is obtained using suitable transformation of the fuzzy variables. Using the proposed inverse reliability method in conjunction with linear and quadratic approximations, the membership functions of A_1^* and A_2^* values at the target reliability index are obtained, and shown in Figs. 2(b) and 2(c). The effect of number of sample points is studied by varying n from 3 to 9 and observed that the membership functions of A_1^* and A_2^* values estimated by the proposed method using $n = 7$ and 9 are overlapping each other.

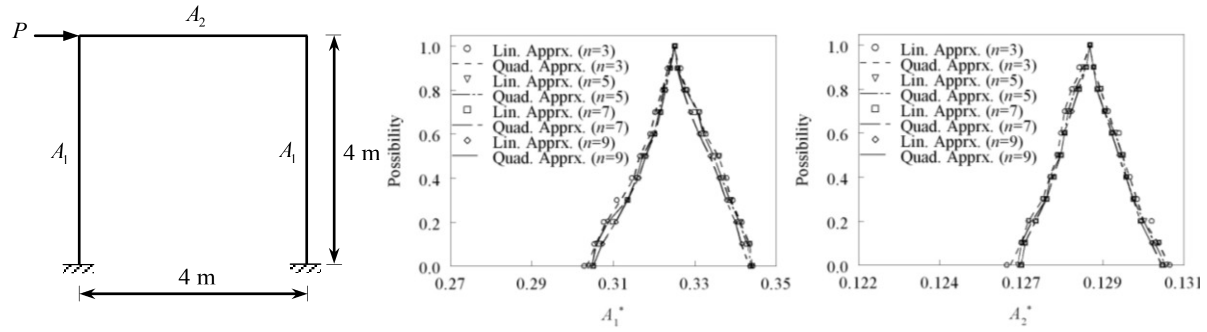

Figure 2: (a) Single story linear frame; (b) A_1^*; and (c) A_2^*

5. Summary and Conclusions

An efficient, accurate, robust solution procedure alternative to existing inverse reliability methods is proposed for nonlinear problems with implicit response functions, that can be used to determine multiple unknown design parameters such that prescribed reliability indices are attained in the presence of mixed uncertain (both random and fuzzy) variables. The proposed method avoids the requirement of the derivatives of the response functions with respect to the uncertain variables. The proposed computational procedure involves three steps: (i) probability of failure calculation using High Dimensional Model Representation (HDMR) for the limit state function approximation, transformation technique to obtain the contribution of the fuzzy variables to the convolution integral, and fast Fourier transform for solving the convolution integral, (ii) reliability index update, and (iii) most probable point (MPP) update. The limit state function approximation is obtained by linear and quadratic approximations of the first-order HDMR component functions at MPP. The methodology developed is versatile, hence can be applied to highly nonlinear or multi-parameter problems applicable involving any number of fuzzy variables and random variables with any kind of distribution. The accuracy and efficiency of the proposed method is demonstrated through six numerical examples.

In addition, a parametric study is conducted with respect to the number of sample points used in approximation of HDMR component functions and its effect on the estimated solution is investigated. An optimum number of sample points must be chosen in approximating HDMR component functions. Very small number of sample points should be avoided as approximation may not capture the nonlinearity outside the domain of sample points and thereby affecting the estimated solution.

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