Concrete Bridge Deterioration Prediction using Markov Chain Approach

Sujeeva Setunge
School of Civil, Environmental and Chemical Engineering, RMIT University
sujeeva.setunge@rmit.edu.au
Md Saeed Hasan
School of Civil, Environmental and Chemical Engineering, RMIT University
m.hasan@student.rmit.edu.au

Abstract

Condition monitoring diagnosis of distress and forecasting deterioration, strengthening and rehabilitation of aging bridge structures is a challenge faced by many road authorities in the world. The accurate prediction of the future condition of bridge elements is essential for optimising the maintenance activities. Most authorities conduct regular condition inspection activities followed by a higher level inspection to diagnose specific distress mechanisms. However, network level modelling utilizing condition data to predict the future condition of bridges is a need identified by bridge asset managers. In developing deterioration models for bridges, one of the major drawbacks is the limited availability of detailed inspection data. Condition data collected using discrete condition rating schemes most of the time are inadequate to develop deterministic deterioration models.

Among the reliability based models which can be derived using limited condition data, Markov models have been used extensively in modelling the deterioration of infrastructure facilities. These models can predict the conditions of bridge elements as a probabilistic estimate. This paper presents an approach used in the prediction of future condition of reinforced concrete and timber bridge elements using a stochastic Markov chain model. Condition data obtained from two local councils in Victoria, Australia has been used in derivation of the models.

Keywords: High volume fly ash concrete, sustainable materials, alkali activated cement

1. Introduction

Various random impact factors such as changes in loading and environmental conditions could initiate time dependent deterioration in Civil infrastructure. Concrete bridge structures exposed to environment are one category of Civil infrastructure, which is prone to rapid deterioration. Influencing factors may range from freeze thaw cycling, traffic wear and tear, exposure to aggressive environments such as sulphate, chloride ions, construction or design errors and inadequate maintenance regimes (Morcous, 2006).

In fact, in most economically developed countries, existing concrete bridge structures are at various stages of deterioration progression and require significant maintenance. At the same time it is easily understandable that due to purely economic reasons, this situation cannot be countered by simply re-building everything anew. Hence, to ensure sustained serviceability and safety of these structures, maintenance interventions become mandatory, which allow partial or complete structural rehabilitation. However, to rationalize decisions with respect to maintenance or rehabilitation, bridge management systems (BMS) are introduced and exercised by many road transport authorities in different countries like in North America, Europe, Japan (Soderqvist and Veijola, 1993, Miyamoto, 2009, DeBrito et al., 1997). The basic procedure of these management systems can be summarized as: (a) assessment of bridge conditions, (b) forecasting of further bridge deterioration, and (c) identification and prioritization of maintenance needs and their corresponding financial requirements.

There are a large number of bridges in Australia that are in conditions of serious deterioration (Stewart, 2001). People in decision making position must decide, when and how to repair, rehabilitate or replace these bridges in order to upgrade service quality. These decisions will change the current performance of these bridges, at the same time also considerably affect their performance and maintenance decisions in the future. Worldwide it is widely recognized that bridge management decisions should be made based on evaluation of life cycle considerations.

1.1 Bridge Deterioration Modelling

Within available resources and a possible set of maintenance and rehabilitation (M&R) actions, the objective of infrastructure management is to determine the best suitable M&R decisions in the current year and in future years. The solution is based on the consequences of possible actions on the expected condition of the system. Since the future condition is indeterminate, deterioration models are used to predict the future condition. This widespread framework is widely used in all existing bridge management optimization methods. However, the actual formulation of the optimization and the deterioration models differ.

Researchers developed different schemes to improve the bridge management decision processes along with the development of BMS. Madanat et al. (1997) introduced an ordered probit analysis to develop an incremental and discrete deterioration model where the difference in observed ratings is an indicator of the underlying latent deterioration. A semi parametric hazard rate model was developed by Mauch and Madanat (2001) to predict times between changes in the condition of concrete bridge decks. According to them the age of structures and other explanatory variables like average daily traffic (ADT), contribute to bridge deck deterioration. In their model they did not included certain important covariant, for example, records of maintenance of bridge decks as the Indiana Bridge Inventory database did not contain all relevant data. Robelin and Madanat (2007) formulated a realistic historical model of bridge deck deterioration using a Markov chain while retaining aspects of the history, namely, deterioration and maintenance, as part of the model to overcome the limitations of the existing Markovian models.

Since the early 1970s several models have been developed (specifically for pavements) to assist decision-makers in predicting the future condition of a network of facilities and, consequently, optimizing the allocation of the network. These models can be grouped into three categories that are not mutually exclusive: deterministic models, stochastic models, and artificial intelligence models. These categories are listed and discussed below,

a) Deterministic Models

Using a mathematical or statistical formulation deterministic models portray relationships between the factors affecting facility deterioration and the condition of the facility. These models calculate the predicted conditions deterministically by ignoring the random error in prediction. Deterministic models can efficiently perform the analysis of networks with a large population. However, they are considered to have the following drawbacks:

- The uncertainty of data due to the inherent stochasticity of infrastructure deterioration and the existence of unobserved explanatory variables is neglected (Madanat et al., 1995, Jiang and Sinha, 1989)
- The current condition and the condition history of individual facilities are not considered while predicting the average condition of a family of facilities (Shahin et al., 1987, Jiang and Sinha, 1989)
- They estimate facility deterioration for the ‘‘no maintenance’’ strategy only because of the difficulty of estimating the impacts of various maintenance strategies (Sanders and Zhang, 1994)
- The interaction between the deterioration mechanisms of different facility components such as between the bridge deck and the deck joints are also not in consideration in these models (Sianipar and Adams, 1997)
- Updating these models with new data is difficult

b) Stochastic Models

The uncertainty and randomness of facility deterioration process are considered as one or more random variables in stochastic models. Among the stochastic techniques Markovian models has been used extensively in modelling the deterioration of infrastructure facilities (Butt et al., 1987, Jiang et al., 1988). These models use the Markov Decision Process (MDP) to determine the expected future condition of facility based on previous condition. MDP is based on the concept of defining states of facility conditions and obtaining the probabilities of facility condition transition from one state to another during one inspection period (Jiang et al., 1988). The uncertainty of the deterioration process and considering the current facility condition in predicting the future one, these two problems of deterministic models has been covered by markovian models. However, they still suffer from the following limitations:

- The discrete transition time intervals, constant bridge population, and stationary transition probabilities assumptions of markovian models are sometimes impractical (Collins, 1974)
- Currently implemented in some advanced BMS (e.g., Pontis and BRIDGIT) markovian models use the first-order MDP that assumes state independence for simplicity (DeStefano and Grivas, 1998), meaning that the future facility condition depends only on the current facility condition and not on the facility condition history, which is unrealistic (Madanat et al., 1997)
- Transition probabilities assume that the condition of a facility can either stay the same or decline, so facilities where treatment actions has been performed cannot be considered for developing transition probabilities (Madanat and Ibrahim, 1995)
- The interaction between the deterioration mechanisms of different facility components are still not efficiently considered in markovian models (Sianipar and Adams, 1997) and
- Transition probabilities require updates when new data are obtained as bridges are inspected, maintained, or rehabilitated, which is a time-consuming task.

c) Artificial Intelligence (AI) Models

Artificial intelligence (AI) models exploit computer techniques that aim to automate intelligent behaviours. AI techniques comprise expert systems, artificial neural networks (ANN), genetic algorithm (GA), and case based reasoning (CBR) to optimise the prediction of future conditions. Sobanjo (Sobanjo, 1997) has performed a detailed investigation to use the ANN in modelling bridge deterioration. A multilayer ANN was utilized to relate bridge age (in years) to the condition rating of the bridge superstructure. A more detailed investigation has been made by Tokdemir et al. (2000) to predict the bridge sufficiency rating using age, traffic, geometry, and structural attributes as explanatory variables. Even though ANN has automated the process of finding the polynomial that best fits a set of data points, it still shares the problems of the deterministic models.

2. Performance Prediction Using Markov-Chain Models

2.1 Markov Chain

Markov chain process can be used to model the deterioration process which has been suggested by many researchers. The basic idea for modelling the deterioration process as a Markov chain process has been provided by Bogdanoff (1978). At early ages many researcher (Golabi et al., 1982, Carnahan et al., 1987) have proposed use of Markov chain model in pavement management systems. A similar approach has been introduced to BMS by Jiang (1990).

To understand the Markov chain consider a set of states, $S = \{s_1, s_2, \ldots, s_T\}$. The process starts in one of these states and moves successively from one state to another. This move is called a step. If the chain is currently in state s_i, then it moves to state s_j at the next step with a probability denoted by p_{ij}, and this probability does not depend upon which states the chain was in before the current state. The probabilities p_{ij} are called transition probabilities. The process can remain in the state it is in which is called holding time, and this occurs with probability p_{ii}. An initial probability distribution, defined on S, specifies the starting state. Usually this is done by specifying a particular state as the starting state.

Markov chain is the distinctive case of the Markov process whose development can be treated as a series of transitions between certain states. Markov process describes the probability of attaining a future state in the process which is dependant only on the present state not on the previous state or how it was attained to that stage (Parzen, 1962). This property can be expressed for a discrete parameter stochastic process (X_t) with a discrete state space as,

$$P\left(X_{t+1} = i_{i+1} \middle| X_t = i_t, X_{t-1} = i_{t-1}, \ldots\ldots, X_1 = i_1, X_0 = i_0\right) = P\left(X_{t+1} = i_{i+1} \middle| X_t = i_t\right) \qquad 1$$

where

it is the state of the process at time t; and

P is the conditional probability of any future event given the present and past events.

While developing performance prediction models for bridge components Markov chains are used., which includes defining discrete condition states and accumulating the probability of transition from one condition state to another over multiple discrete time intervals. Transition probabilities are represented by a matrix of order $n \times n$ called the transition probability matrix (P), where n is the number of possible condition states. Each element ($p_{i,j}$) in this matrix represents the probability that the condition of a bridge component will change from state (i) to state (j) during a certain time interval called the transition period. If the initial condition vector P(0) that describes the present condition of a bridge component is known, the future condition vector P(t) at any number of transition periods (t) can be obtained as follows (Collins, 1975):

$$P(t) = P(0) \times P^t \qquad 2$$

where

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,n} \\ .. & . & \cdots & .. \\ .. & .. & \cdots & .. \\ .. & .. & \cdots & .. \\ P_{n,1} & P_{n,2} & \cdots & P_{n,n} \end{bmatrix} \qquad 3$$

Markov chain theory is established on two fundamental conventions: memoryless and homogeneous. The memoryless rule, sometimes known as the ‘Markov property’, stipulates that the future states of the process depend only on the current states; while the (time) homogeneous rule requires that the rates of transition from one state to another remain constant throughout the time. Implicit in the time-homogeneity assumption of the Markov chain theory is the presumption of geometric distribution (in the case of discrete time) or exponential distribution (in the case of continuous time) for the holding time (Howard, 1971). Holding time is the duration that the process sojourns in one particular state before moving to another. The geometric and exponential distributions possess the "memoryless" property. Used in bridge deterioration modelling the memoryless property of the holding time implies that the rate of leaving a state is constant irrespective of how long a bridge has been in that state.

2.2 Obtaining Transition Probability Matrix

2.2.1 Frequency Approach

The simplest technique adopted to calculate the probability transition matrix from condition data is the percentage prediction method. This approach is quite simplistic and can be obtained directly from the condition data. The probability 'P_{ij}' of transition in bridge element condition from state 'i' to state 'j' can be estimated using the following equation (Jiang et al., 1988),

$$P_{ij} = \frac{n_{ij}}{n_i} \qquad 4$$

Where 'n_{ij}' is the number of transitions from state 'i' to state 'j' within a given time period and 'n_i' is the total number of elements in state 'i' before the transition. Where there is no enough data points to complete the matrix, missing transition probabilities can be derived using a linear relationship as described below (in this case P_{23}),

$$\frac{P_{22}}{P_{33}} = \frac{P_{12}}{P_{23}} \qquad 5$$

2.2.2 Regression Approach

The regression-based optimisation method is the most-commonly used approach in estimating transition matrices for different types of facilities, such as pavements and bridges (Bulusu and Sinha, 1997). This method uses a non-linear optimization function to minimize the sum of absolute differences between the regression curve that best fits the condition data and the conditions predicted using the adopted Markov chain model. The objective function and the constraints of this optimization problem can be formulated as follows (Butt et al., 1987),

minimize $\sum_{t=1}^{N} \| C(t) - E(t) \|$

subject to $0 \le p_{ij} \le 1$ for i, j=1, 2, ….., n 6

$\sum_{i=1}^{n} p_{ij} = 1$

where
N is the total number of transition periods;
C(t) is the facility condition at transition period number t based on the regression curve;

and E(t) is the expected value of facility condition at transition period number t based on Markov chains, which is calculated as follows:

$$E(t) = P(t) \times S \qquad 7$$

where S is the condition states vector.

2.3 Adopting Markov Model in BMS

The probability P defined in Equation 1 is called the transition probability which can be written in short as follows,

$$P\{X_{t_n} = x_n | X_{t_{n-1}} = x_{n-1}\} = p_{x_{n-1}} x_n \qquad 8$$

This is the conditional probability of the system or element being in state x_n at t_n , given that it was in state x_{n-1} at t_{n-1} . This probability is also referred to as the one-step transition probability, since it describes the transition of the condition between times t_{n-1} and t_n, over one time step or one time interval.

For example, p_{34} = 30% for a bridge element means that the probability that this element will be in state 4 at t_n, if it was in state 3 at t_{n-1}, is 30 percent. Here t_n can be, for example, Year 1997, and t_{n-1} Year 1996. This also indicates that the prediction based on the Markov Chain is probabilistic, or with uncertainty taken into account.

Similarly an m-step transition probability is thus defined as

$$p_{x_n x_{n+m}} = P\{X_{t_{n+m}} = x_{n+m} | X_{t_n} = x_n\} \qquad 9$$

Here (n + m)-n = m steps indicating the time difference between t_{n+m} and tn. Each step here can be defined as a day, a month, a year, 2 years, 10 years, etc., depending on the system and its states of interest. For bridge management, Pontis uses a year as a typical time step. Namely the transition probability matrices for bridge elements are implicitly for 1-year periods.

3. Methodology

3.1 Data Collection

For the preliminary proejct, data was sourced from Roads corporation of Victoria. The database includes South Western Victoria bridge inspection data and for the purpose of this research project a relevant data set was extracted to carry out the investigation. This data was carefully selected, with consideration given to size, quality and suitability, to provide the most comprehensive study possible. Data from VicRoads was accessible in a spreadsheet format and included the following information on Bridges:

Structure id	Structure Type Bridge
Road Name	Overall Length (m)
Feature Crossed	Overall Width (m)
No of Spans	Clear Width (m)
Chainage	Traffic Width (m)
Start Reference Point	Date of Level 3 Inspection
Distance Past	Date of Level 2 Inspection
Road Number	Date of Level 1 Inspection
MABC Classification	Load Capacity (Tonnes)
Road Classification	High Mass Limit Constraint Y/N
Region LGA	Monitor Bridge YIN
Latitude	Height ConstraintY/N
Longitude	Other Agency / Responsibility
Year Constructed	AADT
Structure Form	

A dataset first was chosen for the deck/slabs precast concrete component (8P). Thirty of these 8P components from twenty three bridges across South Western Victoria were selected to carry out the bulk of the analysis. Data from components often displayed an improved condition at the components next inspection. This effect is due to rehabilitation and consequently this data was excluded as it cannot be used to calculate the effects of pure deterioration. Other data removed included bridges whose records displayed two inspections occurring on the same date. This could be attributed to human error during data recording. As a result this data is not suitable for deterioration prediction using the Markov model which requires two data points over a time interval. A sample data set of some structures is provided in table 1.

Table 1: Sample Inspection Data (8P)

Structure ID	*Date Constructed*	*First Inspection Date:*	*Inspection Condition Rating*				*Second Inspection Date:*	*Inspection Condition Rating*				*Inspection Interval*
			1	*2*	*3*	*4*		*1*	*2*	*3*	*4*	
SN2086	*30/6/1959*	*20/4/’04*	*100*	*0*	*0*	*0*	*6/06/'06*	*100*	*0*	*0*	*0*	*2.1*
SN3936	*30/6/1952*	*3/4/’03*	*82*	*5*	*11*	*2*	*6/06/’06*	*82*	*5*	*11*	*2*	*3.2*
SN3232	*01/6/1940*	*3/5/’02*	*100*	*0*	*0*	*0*	*24/1/’06*	*78*	*22*	*0*	*0*	*3.7*
SN2800	*30/6/1961*	*1/6/’02*	*100*	*0*	*0*	*0*	*4/04/’06*	*100*	*0*	*0*	*0*	*3.8*
SN2104	*30/6/1963*	*16/4/’04*	*69*	*10*	*21*	*0*	*24/1/’07*	*35*	*44*	*14*	*7*	*2.8*

3.2 Data Analysis

Analysis of the data included a deterministic procedure followed by adopting the Markov prediction model to a range of bridge components. Firstly, the deterministic prediction method was adopted to identify the trend of component condition state. Secondly, the percentage prediction method based

Markov chain was applied to obtain the deterioration trend of the bridge elements. Percentage prediction method was used to obtain the transition probability matrix and thus calculate the future expected conditions of the bridge elements.

4. Results and Discussions

4.1 Deterministic Procedure

Adoption of the deterministic procedure to the bridge inspection data, in order to generate a deterioration curve for a deck/slab precast concrete component (8P), is represented graphically in Figure-1. It is obvious that no trend is occurring with the inspection data as there is a lot of scatter which can be attributed to maintenance actions performed on some components. This is well displayed by the yellow data points which suggest these components have a condition rating of 1, i.e. brand new condition, after 30 years. It is reasonable to assume a certain level of deterioration would occur on components after such time. It was clear that a deterministic method could not be used to forecast deterioration using this condition data.

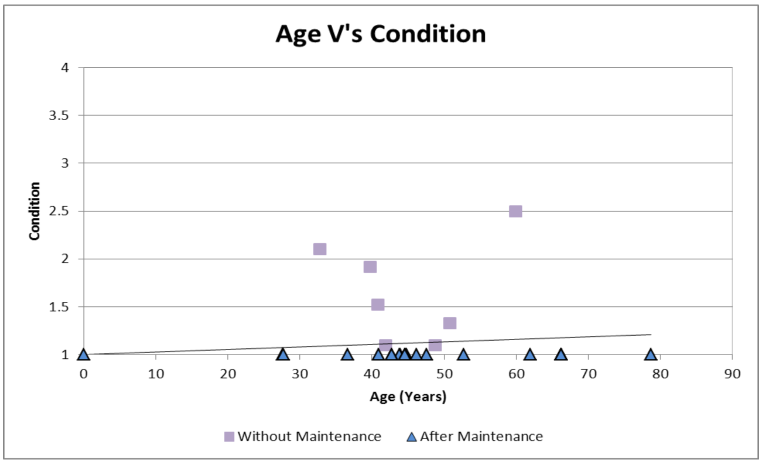

Figure 1: Condition of slabs/decks (8P) with time

4.2 Markov Chain Percentage Prediction Method

Deterioration prediction was carried out using the Markov chain approach as outlined in section 3.2 using the percentage prediction method. The following transition matrix (table 2) was obtained for the deck/slab precast concrete component (8P).

Table 2: Transition Matrix (8P)

	1	*2*	*3*	*4*	*SUM*
1	*0.67*	*0.27*	*0.06*	*0.00*	*1.0*
2	*0.00*	*0.96*	*0.04*	*0.00*	*1.0*
3	*0.00*	*0.00*	*0.93*	*0.07*	*1.0*
4	*0.00*	*0.00*	*0.00*	*1.00*	*1.0*

The future condition of the component was then calculated using equation 2. The initial state vector is assumed to be (1, 0, 0, 0) which represents the probability that 100% of the component will be in condition state 1 at time zero. Also, an average of the difference in inspection periods was used to calculate a time increment, t, of 3.8 years. The component condition after the first time interval, 3.8 years, was calculated as (0.67, 0.27, 0.06, 0.00) and after the second time interval, 7.6 years, as (0.45, 0.44, 0.11, 0.00). The conditions after certain time intervals, i.e. transient probabilities, are shown in Table 3 and a plot is given in Figure 2. Figure 3 displays the expected component deterioration, as calculated using the Markov chain percentage prediction method, over a 100 year period. The expected component condition is determined from the calculated transient probabilities using the following equation,

$$C_e^t = (1P_1^t + 2P_2^t + 3P_3^t + 4P_4^t) \qquad 10$$

where, t is the time in years.

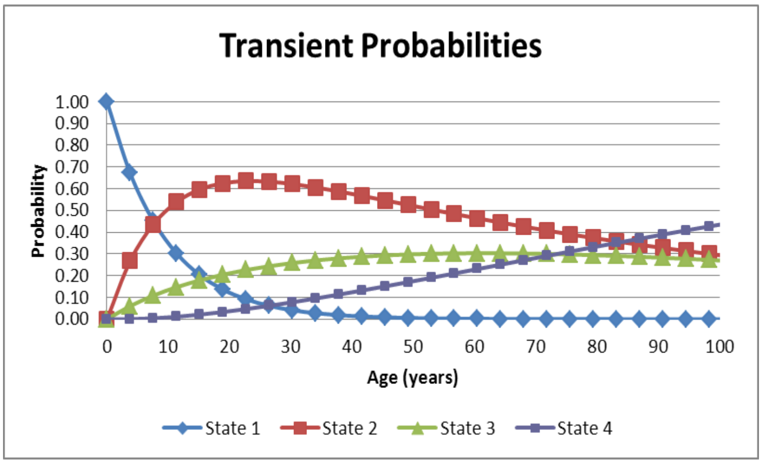

Figure 2: Transient probabilities for slabs/decks (8P)

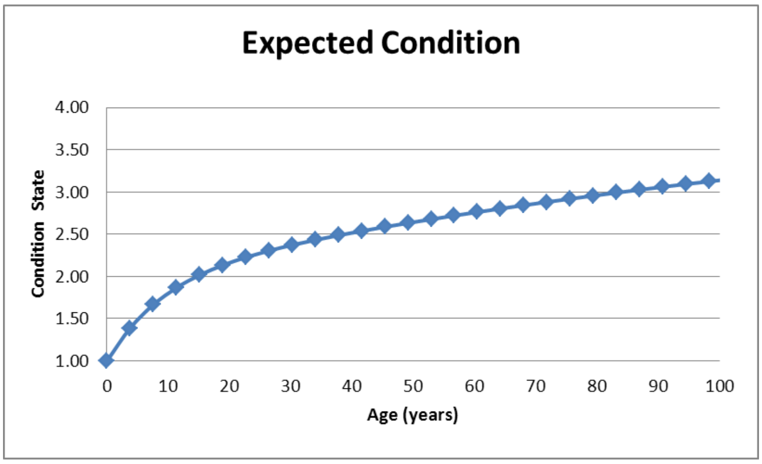

Figure 3: Expected conditions for slabs/decks (8P) using Markov model

An initial high deterioration rates up to first 20 years for the bridge slabs/decks component is observed from Figure-3. Afterwards the rate of deterioration gradually decrease over time until it reaches to condition state 3, which is after 80 years of construction. The Markov chain percentage prediction method was also applied to determine deterioration rates for three other components namely diaphragm in-situ concrete, column/pile concrete, and abutment concrete. Figure-4 compares the predicted rate of deterioration for deck/slab precast concrete, diaphragm in-situ concrete, column/pile extension in-situ concrete and abutment in-situ concrete components. The results demonstrate that the deck has the highest rate of deterioration followed by abutments, diaphragms and columns. The curves represent what could be expected as the rate of deterioration for these components continues without any maintenance action undertaken. However, a higher level of deterioration perhaps would have been expected for the column component which is most likely to have a more aggressive exposure due to possible contact with water, tidal and splash zones, spray and also soils with aggressive chemicals.

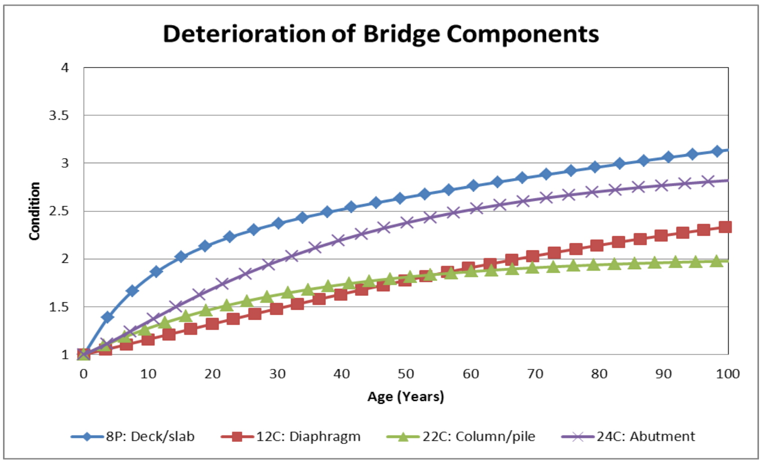

Figure 4: Comparison of expected deteriorated condition of different components

5. Conclusions

For the condition data collected, a deterministic prediction method will not be appropriate for modeling the deterioration of bridge components. The Markov chain approach appears to offer a superior solution by using the percentage prediction method to develop the transition matrix. Using the developed transition matrices, some preliminary conclusions about deterioration of the bridge components can be made. Based on the data available for this investigation, deck/slabs appear to have a faster deterioration compared to other components followed by abutments, diaphragms and piles.

A better outcome by means of calculating more accurate deterioration models would be possible if records of maintenance action were available, the inspection period was constant and more inspection data was available.

6. Recommendation for Future Work

From this investigation it has been established that, with refinement, the use of Markov chain approach for deterioration prediction of bridge components is a suitable model. The Markov model could be implemented into current bridge management systems in order to assist in maintenance and replacement decisions amongst road authorities and local councils in Victoria based on VicRoads bridge inspection data. The following list recommends further investigation into the Markov chain model for deterioration prediction of bridge components,

- Collecting data from other regions across Victoria to expand the database and improve accuracy of the Markov model.

- Investigate adjusting the transition matrix to simulate a constant inspection period i.e. implement Bayes' rule to the transition matrices.
- Investigate the interaction between components during progression of deterioration.
- Develop deterioration models for all bridge components and apply a weighting by means of importance of each component to generate an overall bridge condition rating.

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