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APPLICATION OF META-MODELING BASED FINITE ELEMENT SOLUTION CONVERSION METHOD TO DEVELOP SMART INITIAL GUESS OF CONJUGATE GRADIENT METHOD FOR SOLID ELEMENT SIMULATION

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Abstract: Smart initial guess of Conjugate Gradient (CG) method for solid finite element simulation is developed by using the meta-modeling based finite element solution conversion method. The key feature of this solution conversion method is the rigorousness; the meta-modeling ensures that the most appropriate structural or solid element solution is the one that is close to the solid or structural element solution, respectively, since an error between structural and solid element solutions is accurately defined in a solution space of continuum mechanics. The initial guess of CG method of this study is developed by using an approximate solid element solution which is converted from a relevant structural element solution. Two numerical examples; a cantilever beam and frame problems, of the proposed CG method are presented. The results show that the computational cost is significantly reduced in the proposed CG method as compared to the ordinary CG method. There is an expectation that this reduction of computational cost of solid element simulation will become more significant with the size of the problem targeted.

Keywords: continuum mechanics; conjugate gradient method; finite element method; structural mechanics

1. Introduction

In civil engineering, the use of a structural element, such as truss, beam, plate or shell is standard. It is rare that a solid element analysis is made for a structure of complicated configuration. While there are several reasons for this rare use of solid element analysis, one of the major reasons is the higher computational cost of solid element analysis than that of structural element analysis. The computational cost of solid element analysis can be reduced by introducing smart improvements for current solid element solvers such as Conjugate Gradient (CG) method [1-6]. The CG method can be improved by introducing a good initial guess or a proper preconditioning.

In order to relate structural and solid models which are constructed for one structure, the authors are proposing meta-modeling [7-12]. As for mechanical response, meta-modeling starts from continuum mechanics as the basic physics, and allocates continuum mechanics modeling as the most accurate modeling. By adding mathematical approximations, meta-modeling derives another modeling such as beam model or shell model. The

Key concept of meta-modeling is that all modelings solve the same physical problem of continuum mechanics but uses different mathematical approximations.

In this study, physics-based initial guess of CG method for solid element analysis to reduce the number of iteration loops that is needed to reach target accuracy is studied. The meta-modeling theory which allocates structural mechanics as a mathematical approximation of continuum mechanics, is used as a candidate to develop this physics-based initial guess of CG method. Based on meta-modeling, we can readily classify the finite element analysis. That is, solid element analysis is the most accurate, and the structural element analysis is an approximation of the solid element analysis.

The content of this paper is as follows. First, the theory of meta-modeling for structural models are briefly explained by using the beam model in Section 2. Then, in Section 3, conversion of a structural element solution to an initial guess of a solid element solution for the CG method is presented. We then carry out numerical experiments to check the performance of the developed method in Section 4. Concluding remarks are made in Section 5.

2. Meta-modeling theory for beam model

Meta-modeling theory introduces equivalent Lagrangian instead of ordinary Lagrangian for velocity, strain and stress, i.e.,

$$\mathcal{L}^*[\boldsymbol{v},\boldsymbol{\epsilon},\boldsymbol{\sigma}] = \int_V \left(\frac{1}{2}\rho\boldsymbol{v}\cdot\boldsymbol{v} - \left(\boldsymbol{\sigma}:\boldsymbol{\epsilon} - \frac{1}{2}\boldsymbol{\sigma}:\boldsymbol{c}^{-1}:\boldsymbol{\sigma}\right)\right) \mathrm{d}V, \quad (1)$$

where, V, ρ, $\boldsymbol{c}$, $\boldsymbol{v}$ and $\boldsymbol{\epsilon}$ respectively are, volume of the body, density, isotropic elasticity tensor, velocity and strain. Further, $\boldsymbol{v} = \dot{\boldsymbol{u}}$ and $\boldsymbol{\epsilon} = \mathrm{sym}\{\boldsymbol{\nabla u}\}$ with $\dot{(\cdot)}$ and $\boldsymbol{\nabla}(\cdot)$ being temporal derivative and gradient, $\cdot$ and $:$ are the inner product and second-order contraction respectively and sym stands for the symmetric part. Considering the meta-modeling theory, the governing equation of beam theory can be obtained from $\mathcal{L}^*$ of Eq. (1), by considering $\{\boldsymbol{u},\boldsymbol{\sigma}\}$ of the following non-zero components:

$$u_1 = -zw'(x,t), \qquad u_3 = w(x,t),$$

$$\sigma_{11} = zs(x,t).$$

Here, the x_1- and x_3- axes are the longitudinal and transverse (bending) directions, and x and z are used instead of x_1 and x_3. The bending moment produced by σ_{11} acts around the x_2-axis or y-axis and prime stands for derivative with respect to x. Note that w and s are functions of x and t.

Since $\mathcal{L}^*$ becomes a function of w and s, the variation of $\mathcal{L}^*$ with respect to these functions is

$$\delta\mathcal{L}^* = \int \left(\left(\delta w\rho(\ddot{w}^2 - z^2\ddot{w}'' + z^2 s'')\right) + \delta s z^2\left(\frac{s}{E} + w''\right)\right) \mathrm{d}V. \quad (2)$$

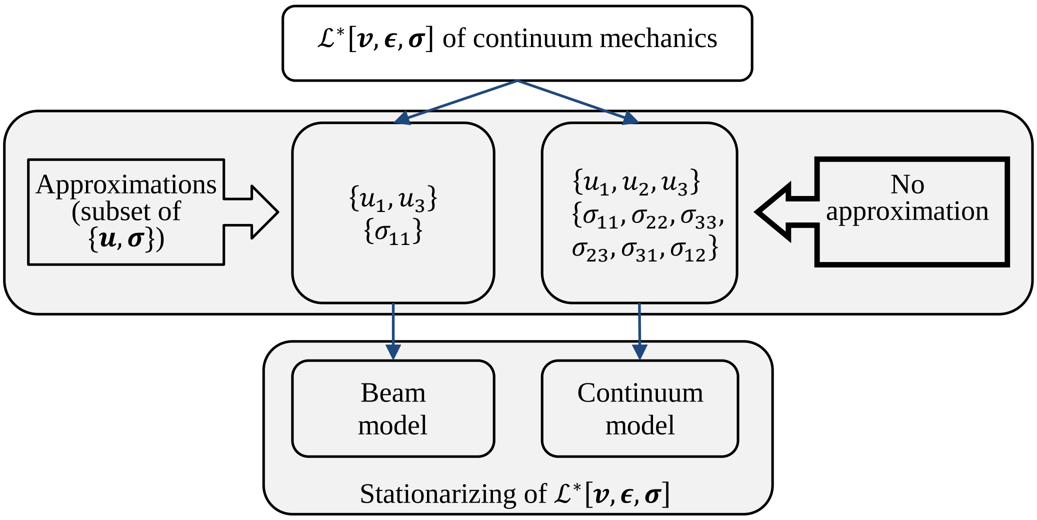
Recall that $\boldsymbol{c}$ is assumed to be homogeneous and isotropic and dot stands for the derivative with respect to t. It thus yields $s = -Ew''$ and

$$\rho A\ddot{w}^2 - \rho I\ddot{w}'' + EIw'''' = 0, \quad (3)$$

where, $A = \int \mathrm{d}z\mathrm{d}y$ and $I = \int z^2\,\mathrm{d}z\mathrm{d}y$; note that $s = -Ew''$ is directly derived from $\delta\int\mathcal{L}^*\,\mathrm{d}t = 0$, and we do not have to make any assumption to relate w and s. Fig. 01 shows graphic view of the meta-modeling theory for beam model.

3. Physics-based initial guess of conjugate gradient method

Based on meta-modeling, a beam element solution is regarded as an approximate numerical solution of a variational problem of $\mathcal{L}^*$ of Eq. (1); see Fig. 02. It is natural to make the conversion from a beam element solution to a solid element solution [8, 11]. Hence, based on meta-modeling theory the beam element solution can be used as a new initial guess of the CG method of solid element analysis. We have not found any study which seeks to apply physics like meta-modeling to get the initial guess for CG method. The basic idea of this study is to use a solid element solution converted from a beam element solution as an initial solution of the CG method.



Note: u_1, u_2 and u_3 are displacement components in the x-, y- and z-directions, respectively, with (x, y, z) being the Cartesian coordinate.

Fig. 01. Meta-modeling for beam model.

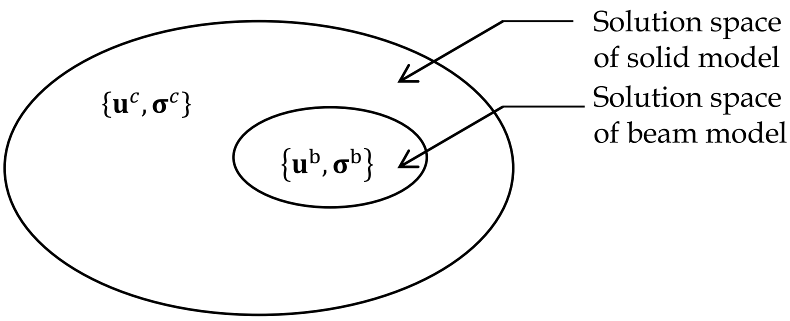


Fig. 02: Solution spaces for beam and solid models; solution space of beam model is subset of solution space of solid model.

To explain the above idea clearly, we denote by $(\boldsymbol{u}^{\mathrm{b}}, \boldsymbol{\sigma}^{\mathrm{b}})$ and $(\boldsymbol{u}^{\mathrm{c}}, \boldsymbol{\sigma}^{\mathrm{c}})$ the beam element solution and the converted solid element solution, respectively. A L2 norm is used as distance in the function space, we define

$$N(\boldsymbol{u}^{\mathrm{c}},\boldsymbol{\sigma}^{\mathrm{c}}) = \frac{|\boldsymbol{u}^{\mathrm{b}}-\boldsymbol{u}^{\mathrm{c}}|^2}{|\boldsymbol{u}^{\mathrm{b}}|^2}+\frac{|\boldsymbol{\sigma}^{\mathrm{b}}-\boldsymbol{\sigma}^{\mathrm{c}}|^2}{|\boldsymbol{\sigma}^{\mathrm{b}}|^2}, \quad (4)$$

where, $|\cdot|^2$ is the L2 norm of $\boldsymbol{u}^{\mathrm{b}}$ and $\boldsymbol{\sigma}^{\mathrm{b}}$ which is calculated by integrating vector or tensor norm of $\boldsymbol{u}^{\mathrm{c}}$ or $\boldsymbol{\sigma}^{\mathrm{c}}$, over V, i.e.,

$$|\boldsymbol{u}^{\mathrm{b}}|^2 = \int_V \boldsymbol{u}\cdot\boldsymbol{u}\,\mathrm{d}V,$$

$$|\boldsymbol{\sigma}^{\mathrm{b}}|^2 = \int_V \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\,\mathrm{d}V\,.$$

We can replace terms of Eq. (4) by generalized nodal displacements, then by minimization process of Eq. (4) with respect to $\boldsymbol{u}^{\mathrm{c}}$, we can obtain an approximate solid element solution for a target problem.

Since $\boldsymbol{u}^{\mathrm{b}}$ is regarded as an approximate solution of the continuum mechanics problem, it is expected that $\boldsymbol{u}^{\mathrm{c}}$ which is converted from $\boldsymbol{u}^{\mathrm{b}}$ will serve as a good initial solution of the CG method. This process is briefly explained in Appendix A.

4. Numerical examples

4.1 Problem setting

Cantilever and frame problems with displacement boundary conditions are discussed under this section to cover applications of the meta-modeling based initial guess for the CG method. Here, the cantilever and frame problem are constructed using beam elements and solid element solution are obtained from the proposed CG method. These numerical experiments show the implementations of the meta-modeling based initial guess for the CG method with its advantages.

Fig. 03 and 04 show the problem settings of these numerical experiments with material data. Only one displacement boundary condition is located at the top left end of frame system; that is 10 mm along x-direction and the bottom of frame is fully fixed. In the cantilever system, three displacement boundary conditions are located at the right end of cantilever system; these are -5 mm, 5 mm and -10 mm along x-, y- and z-directions respectively and the left end of cantilever beam is fully fixed.

4.2 Results and discussion

The initial guesses for the target solid element problems which are shown in Fig. 03 and 04, are constructed from the equivalent beam element systems by minimizing the expression in Eq. (4).

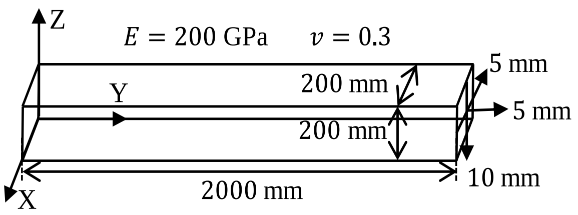


Fig. 03: Schematic view of cantilever beam.

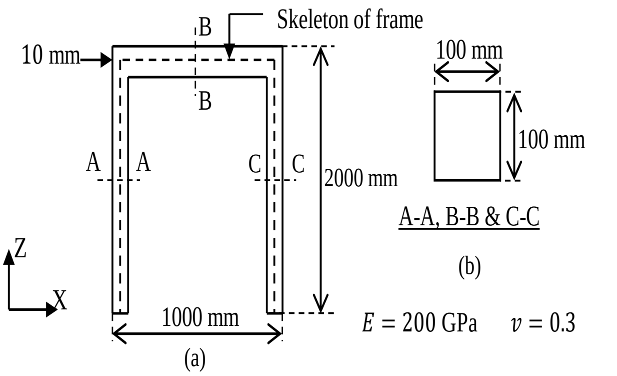
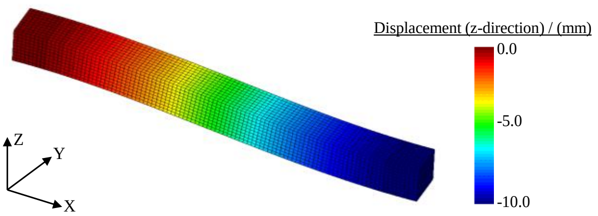


Fig. 04. Schematic view of frame: (a) cross-section of frame; and (b) cross-section of A-A, B-B & C-C.

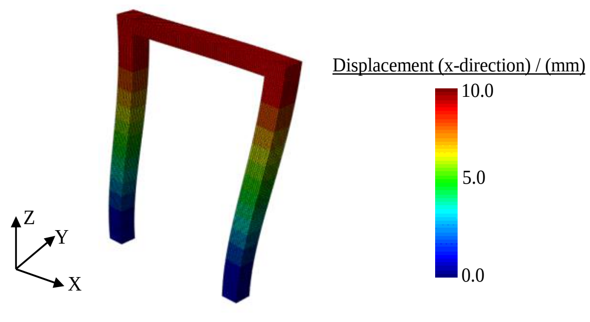
The converted solid element displacement solutions are shown in Fig. 05(a) and 05(b). Performance of the proposed CG method is compared with the ordinary CG method that uses initial guess as zero vector. It shows that, the proposed CG method is 2.62 and 1.92 times faster than the ordinary CG method for the cantilever and frame problem which includes around 50,000 and 80,000 degrees of freedom (dof) respectively. In this example λ is fixed to 1×10^{-8}; see Appendix A for definition of λ. Fig. 06(a) and 6(b) show relative residual in each iteration for both the CG methods.

According to this result, it is clear that the amount of iterations drastically reduces in the proposed CG method as compared to the ordinary CG method; see Fig. 6(a) and 6(b). We hope this positive effect may become larger with a target problem size (dof). If we can create preconditioning from the meta-modeling theory, it is more effective than the meta-modeling based initial guess. Currently, we are working on developing the meta-modeling based preconditioning for the CG method.

5. Concluding remarks

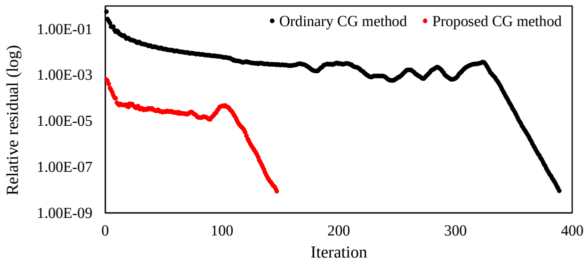


(a)

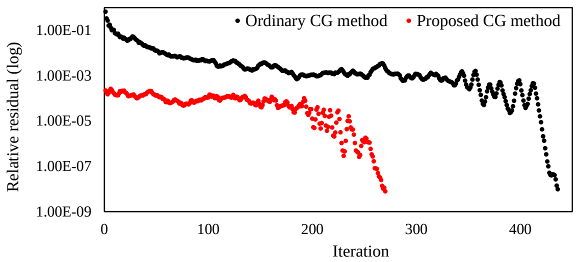


(b)

Fig. 05. Approximated displacement for solid element system from equivalent beam element system: (a) cantilever problem (z-direction) and (b) frame problem (x-direction).



(a)



(b)

Fig. 06. Relative residual in each iteration for both CG methods: (a) cantilever problem and (b) frame problem.

In this paper, physics-based initial guess of CG method for solid element analysis to reduce the number of iteration loops that is needed to reach target accuracy is proposed and it is successfully tested with a frame model. Further investigation is needed to ensure this possibility. Practical problems will be studied to this end. This method needs to be tested with two-dimensional structures such as plate and shell, too.

In the viewpoint of computational mechanics, the use of structure element solution as preconditioning of solver that is used by solid element analysis seems interesting as well as important. While there are numerous mathematical studies about preconditioning, as far as the authors have studied, some possibility is found about physics-based preconditioning by employing meta-modeling theory, and authors are presently working on it.

Appendix A Algorithm 1.

This algorithm is detailed below for solving $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is a real, symmetric, positive-definite matrix. The input vector (initial guess) $\mathbf{x}_0$ is constructed from equivalent beam system and λ is expected relative residual value of iteration.

$$\mathbf{r}_0 := \mathbf{b} - \mathbf{A}, \qquad \mathbf{p}_0 := \mathbf{r}_0, \qquad k := 0.$$

For $k := 1, \dots \dots, n-1$

(n is maximum number of iterations)

$$\alpha_k := \frac{\mathbf{r}_k^T \mathbf{r}_k}{\mathbf{p}_k^T \mathbf{A} \mathbf{p}_k}$$

$$\mathbf{x}_{k+1} := \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$\mathbf{r}_{k+1} := \mathbf{r}_k - \alpha_k \mathbf{A} \mathbf{p}_k$$

if $||\mathbf{r}_{k+1}||_2 / ||\mathbf{b}||_2 > \lambda$ then exit loop

$$\beta_k := \frac{\mathbf{r}_{k+1}^T \mathbf{r}_{k+1}}{\mathbf{r}_k^T \mathbf{r}_k}$$

$$\mathbf{p}_{k+1} := \mathbf{r}_k + \beta_k \mathbf{p}_k$$

the result is $\mathbf{x}_{k+1}$.

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