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NUMERICAL SIMULATION OF PREDICTION OF SHEAR STRENGTH OF REINFORCED CONCRETE BEAM USING TOTAL CRACK STRAIN MODEL

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Abstract: Despite significant experimental, numerical and analytical research, the shear behaviour of reinforced concrete members remains one of the least understood mechanisms in reinforced concrete. Due to the complexity of shear behaviour, empirical or semi-empirical analysis approaches have typically been developed and these are widely employed in codes of practice. With the development of concrete construction industry, now it is common in construction of reinforced concrete moment resisting frames that some columns supported on beams as floating columns resulting a shorter shear span to depth ratio to beams. Furthermore, longer spans as well as shorter spans are in a single frame of multi bays to get the architectural appearance. However, the beam section designed for a longer span is continued even in the shorter span of the frame resulting shorter span to depth ratio in shorter bays. In the design stage of such elements, as a consequence that less attention paid in predicting the shear capacity than moment capacity, the brittle failures mode of beams in shear is observed before the ductile failure mode in moment. This actually violates the concept of ultimate limit state design. Therefore, the objective of the research study is to predict the shear strength of reinforced concrete beams using the total crack strain constitutive model and to validate the prediction with available experimental data in the literature. Simply supported beams are modelled with Midas FEA using Total crack strain model and their results are compared with the experimental results. Then the validated model is used to predict the shear strength of beams in monolithic construction. It was concluded that when predicting the shear failure of reinforced concrete members by total crack strain model, results were very sensitive to the defined shear stress strain relationship.

Keywords: reinforced concrete; shear strength; stresses; stiffness

1. Introduction

Nowadays, it is common in construction of reinforced concrete moment resisting frames that some columns supported on beams as floating columns resulting a shorter shear span to depth ratio to beams. Furthermore, longer spans as well as shorter spans are in a single frame of multi bays to get the architectural appearance. But the beam section designed for a longer span is continued even in the shorter span of the frame resulting shorter span to depth ratio in shorter bays. In such cases, the failure to accurately predict the shear capacity in the design stage of such elements could lead to catastrophic brittle failure of structures in shear as opposed to the preferred ductile failure in moment. This actually violates the concept of ultimate limit state design.

Shear transfer actions and mechanisms in concrete beams are complex and difficult to clearly identify. Complex stress redistributions occur after cracking, and those redistributions have been shown to be influenced by many factors. Pang and Hsu (1995) [1] and Collins et. al. (1996) [2] imposed different levels of relative importance to the basic mechanisms of shear transfer.

Morsch (1902) [3] derived shear stress distribution for a reinforced concrete beam containing flexural cracks. Morsch predicted the shear stress would reach its maximum value at the neutral axis and would then remain constant from the neutral axis down to the flexural steel. Truss models were widely used to understand the shear behavior of reinforced concrete beams in the early 1900's. Ritter (1899) [4] was the first to use a 45^0 truss model for the analysis of the post cracking behavior of a reinforced concrete beam. In that model, diagonal concrete struts were considered to be the diagonal members of the truss, the stirrups were the vertical members of the truss, the longitudinal reinforcement served as the bottom chord of the truss, and the flexural compression zone served as the top chord of the truss. Kani (1964) [3] attempted a more realistic approach by addressing the problem of the bending of the "teeth" of the concrete between flexural cracks. The concrete between two adjacent flexural cracks was considered to be analogous to a tooth in a comb. The concrete teeth were assumed to be free cantilevers fixed in the compression zone of the beams and loaded by the horizontal shear from bonded reinforcement. Although this theory did not cover most of the shear transfer mechanisms, it was probably the start of more rational approaches.

Vecchio and Collins (1986) [5] have introduced an analytical model which is capable of predicting the load-deformation response of a reinforced concrete element that is subjected to in-plane shear and normal stresses. In this model cracked concrete is treated as a new material with its own stress-strain characteristics. Selby and Vecchio (1993) [6] stated that analysis models for concrete cracking can be classified into a discrete crack model and a smeared crack model. The discrete crack model uses finite elements at which concrete cracks are separately represented as boundaries. In the smeared crack model, concrete cracks are assumed to be scattered and distributed, such that discrete elements are not used at the crack location.

Total crack strain constitutive model uses the smeared crack approach in predicting the response of reinforced concrete elements with three uniaxial material models for tension, compression and shear. It is widely used in predicting the flexural response of reinforced concrete elements accurately incorporating moment axial interaction, effects of lateral confinement etc. However, it is identified that the accurate prediction of the shear capacity and the overall load-deflection response of the reinforced concrete element dominated by shear is very sensitive to the shear stress-strain curve used in the constitutive relationship.

Therefore, objective of this study is to validate the capability of Total crack strain model in predicting the shear capacity of reinforced concrete elements with experimental data. Furthermore, this study investigates the effect of boundary condition on the shear capacity of the reinforced concrete elements.

2. Total Crack Strain Model

Analysis models for concrete cracking can be classified into a discrete crack model [discontinuum model] and a smeared crack model [continuum model]. The discrete crack model uses finite elements at which concrete cracks are separately represented as boundaries. In the smeared crack model, concrete cracks are assumed to be scattered and distributed, such that discrete elements are not used at the crack locations. The smeared crack model assumes that locally generated cracks are evenly scattered over a wide surface. This model is known to be suitable for reinforced concrete structures with reasonable amount of reinforcement, and its finite element modelling is relatively simple [5]. The smeared crack model can be classified into orthogonal and non-orthogonal crack models depending on the assumption of angles of crack development. The orthogonal crack model assumes orthogonal crack directions, whereas the non-orthogonal crack model assumes non-orthogonal directions of cracks. Also, depending on the numerical analysis methods for cracks, the smeared crack model is further classified into various models such as a decomposed-strain model and a total strain model.

The decomposed-strain model in the smeared crack model calculates the total strain in terms of material strain and crack strain.

The total strain model in the smeared crack model can be rather simply formulated using total strain without having to decompose it into the strain components.

MIDAS [7] uses the total strain crack model classified under the smeared crack model. It provides two methods, which are separated into the fixed crack model and the rotating crack model depending on the reference crack axes. The former assumes that the axes of cracks remain unchanged once the crack axes are defined. On the contrary, the latter is a method in which the directions of the cracks are assumed to continuously rotate depending on the changes in the axes of principle strains.

An iterative scheme is used for concrete crack analysis because of its nonlinearity. In order to satisfy the equilibrium between external and internal force vectors, one of the incremental iterative procedures such as the Newton-Raphson method is used. To this end, the constitutive model needs to be defined by a proper stiffness matrix. MIDAS uses the secant stiffness and tangent stiffness approaches to determine the stiffness matrix. The secant stiffness approach is especially suitable for finding excellent and stable solutions to analyses of reinforced concrete structures, which widely develop cracks. On the contrary, the tangent stiffness approach is known to be very appropriate for analyses of local cracking or crack propagation. The secant approach is used according to the stiffness of an orthotropic material with zero Poisson's ratio in all directions.

In this study, Total Crack strain model with configuration of fixed crack model including secant stiffness, lateral crack effect and confinement effect was used.

The compression behavior and tension softening of reinforced concrete material are represented by Thorenfeldt and Hordijk models as shown in figures 01 and 02 respectively. It is important to note that the shear stress-strain relationship used in this study is derived by using the Modified Compression Field Theory for a membrane element.

Two simply supported beams called specimen 01 and 02 having span to depth ratio of 2.1 and longitudinal reinforcement percentage of 1.8 were selected from an

3. Finite Element Model Development

3.1 Modelling of simply supported beams

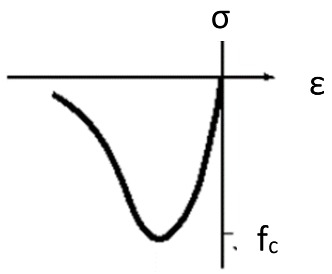

Figure 01: Thorenfeldt compression model

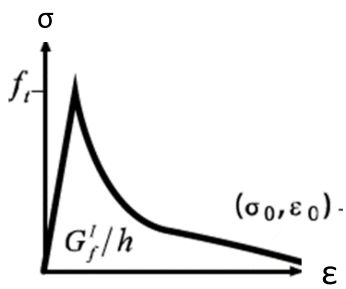

Figure 02: Hordijk tension model

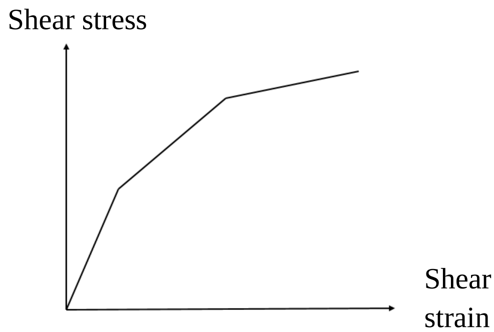

Figure 03: Trilinear shear curve

experimental programme carried out at structural laboratory. The percentage of shear reinforcement is 0.3 for specimen 01 and it is 0.8 for specimen 02. The dimensions of the two specimens were same having 1700 mm in length, 350 mm in depth and 200 mm in width.

In this study, the finite element models of the selected beams were developed in MIDAS FEA programme. Solid elements were used to model the beam and reinforcement elements were used to model longitudinal and transverse reinforcement as shown in Figure 04. Von-mises yield criteria was used for steel material and Total Crack strain model was used for concrete material. The total crack strain model uses fixed crack model with secant stiffness matrix considering lateral crack effect and confinement effect.

The tension softening and compression behaviour of reinforced concrete material are represented by Hordijk and Thorenfeldt models, respectively. It is important to note that the shear stress-strain relationship used in this study is derived by using the Modified Compression Field Theory for a membrane element. Nonlinear static analysis is performed to obtain the load deflection response of the selected beams by using displacement based load control with 0.1 mm increments rather than the force based load control. The displacement increment is applied at the middle of the beam span as in the experiments.

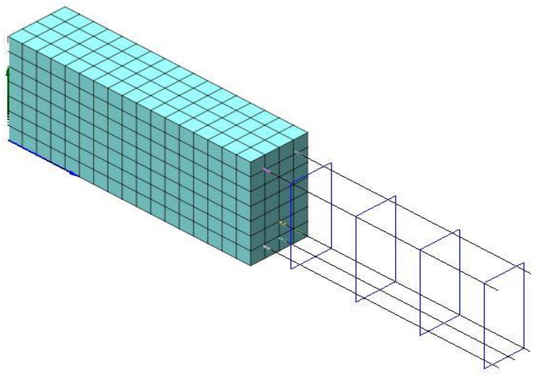

Figure 04: Model of simply supported beam

3.2 Modelling of concrete frame structure

After validating the model, it was used to investigate the effect of support conditions on shear strength of beams in monolithic construction. Same beams were used to model frames selected for this study. But the column dimensions and the reinforcement details were selected such that the moment capacity of reinforced concrete columns are greater than the moment capacity of reinforced concrete beam section. Each beam section was modeled with columns having shear reinforcement percentage of 0.31. Longitudinal reinforcement percentage of column specimen was 1.4 with a section of 300*300mm and having 4*T20 bars. Height of columns were taken as 3000mm.

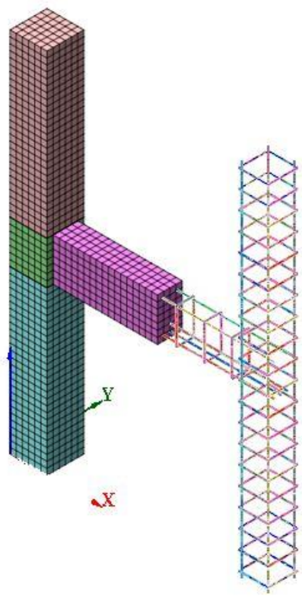

Figure 05: Model of frame structure

4. Results and discussion

4.1 Shear stress-strain curve

Shear stress-strain curve is derived for each specimen used in study by using Modified Compression Field Theory. It is identified that the grade of concrete, the percentage of longitudinal and transverse reinforcement are the key factors affecting the shear stress-strain relationship. Figure 06 illustrates the resultant shear stress-strain curve for specimen 01. It is a trilinear curve indicating that the elastic limit of shear stress is 2.3 MPa.

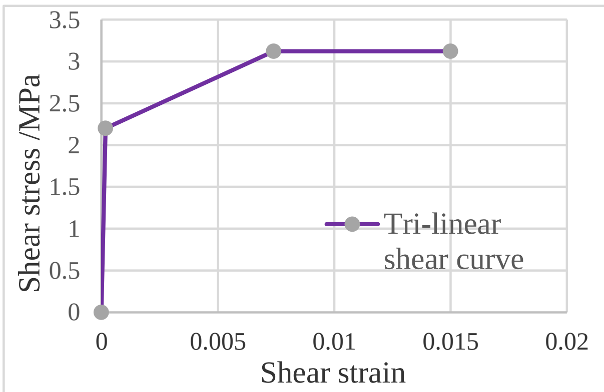

Figure 06: Shear stress strain relationship for specimen 01

4.2 Comparison of experimental and numerical predictions of simply supported beams

Figure 07 illustrates the resultant distribution of principle tensile strain at the peak resisting loads in specimen 01 and 02. Highly concentrated principle strain at the mid depth near the support as shown in Figure 07 indicates the initiation of shear crack and its direction of propagation under the peak resisting load.

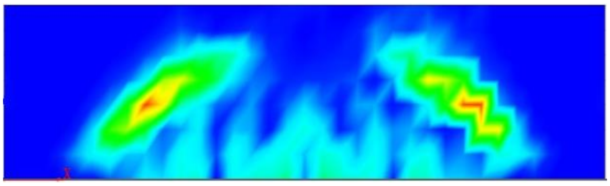

Figure 07: Principal strains at failure

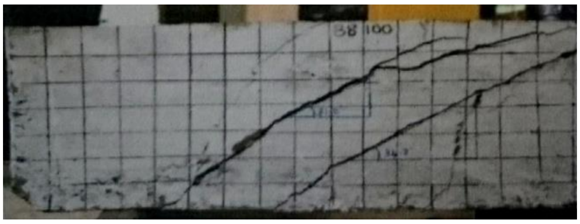

Figure 08: Experimental crack pattern

The angle of inclination of the diagonal cracks in the specimens 01 and 02 are about 45^{0}, in referring the principle strain directions. Figure 09 and 10 illustrates the comparison of experimental and numerical load deflection curves for specimen 01 and 02 respectively. It is clear from the load deflection curves that shear strength of the two specimens are predicted accurately.

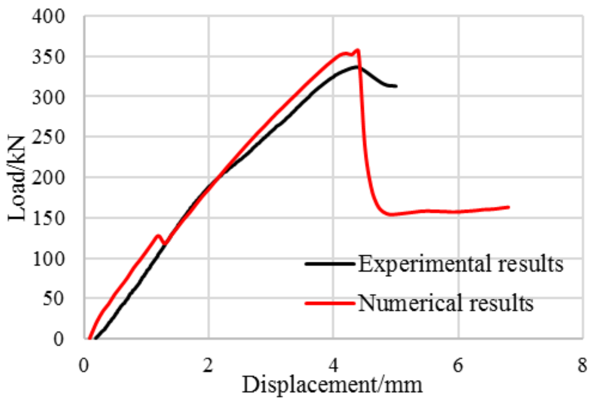

Figure 09: Comparison of experimental and numerical results of specimen 01

When considering specimen 01, peak resisting load and the displacement at peak load are almost the same in numerical and experimental results. Even though peak resisting load is predicted well for specimen 2 post peak response of load deflection is significantly different.

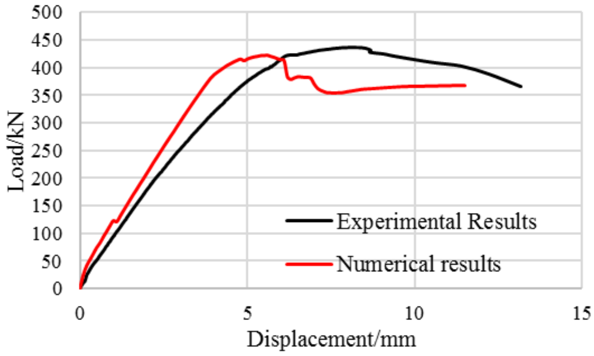

Figure 10: Comparison of experimental and numerical results of specimen 02

Figure 11 shows the reinforcement stresses at failure of the beam. Initially bottom reinforcement of the simply supported beam gets subjected to tensile stresses. But with the increment of load, shear reinforcement yields and beam fails in shear.

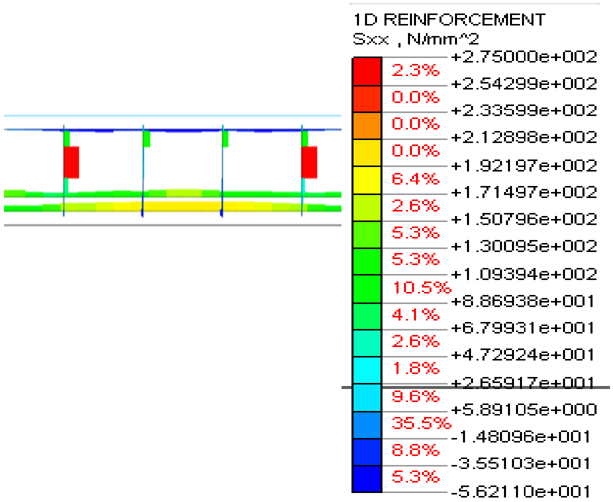

Figure 11: Reinforcement stresses at failure

4.3 Effects of support condition on the shear strength of a beam

Figure 12 and 13 illustrates the comparison of load deflection curves for beam specimen 01 with simply supported condition and in frame with monolithic connection with column. It is clear from the load displacement curves that there is an increment of peak resisting load in joints when comparing with the corresponding beams.

When comparing the direction of principle strains of the beam and corresponding joint, variation of crack angle can be observed.

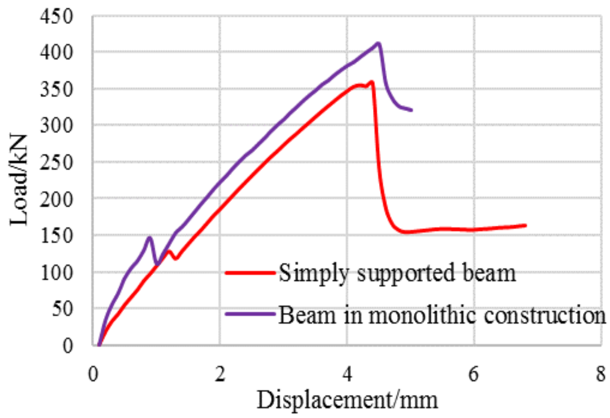

Figure 12: Result comparison of joint specimen 01

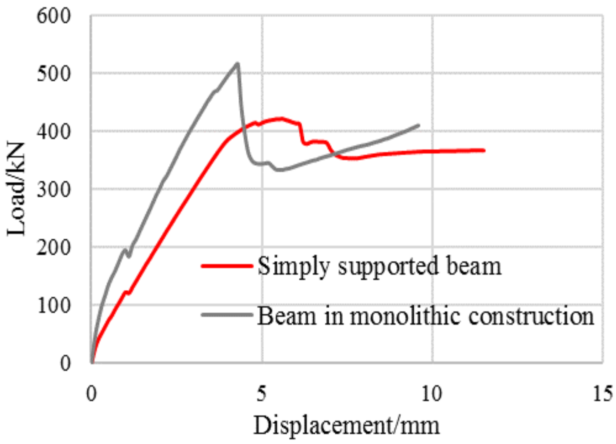

Figure 13: Result comparison of joint specimen 02

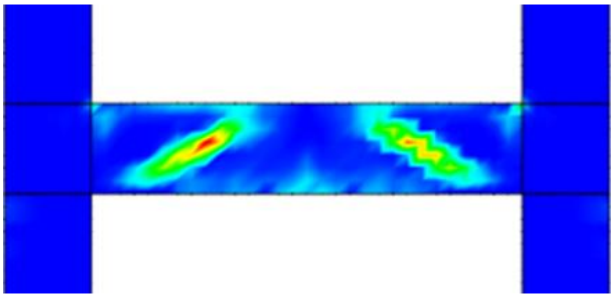

Figure 14: Principal strains of the joint at failure

Figure 15 illustrates yielding of shear reinforcement before yielding the longitudinal reinforcement.

5. Conclusions

When predicting the shear failure of reinforced concrete members by total crack strain model, results were very sensitive for the defined shear stress strain relationship. However, by comparing with experimental

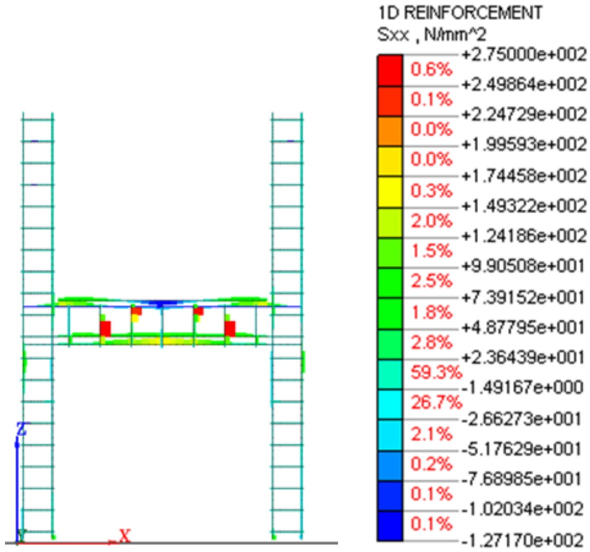

Figure 15: Reinforcement stresses at failure

data, it can be concluded that Total Crack strain model with configuration of fixed crack model including secant stiffness, lateral crack effect and confinement effect and shear stress strain curve derived by modified compression field theory can accurately predict the shear strength, location of cracks and initial stiffness of reinforced concrete beams.

There is about 20% increment of shear strength when comparing the load displacement curve of beam column joint with the load displacement curve of corresponding beam. The main reason for this increase is the moment shear interaction in the beam column joint

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