NANOBEAM MECHANICS: THEORY AND EXPERIMENTAL COMPARISON

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ABSTRACT

Various experimental techniques have been used to determine the mechanical properties at the nanoscale, namely, bending tests, nanoindentation tests, resonant excitation tests, etc. [1-3]. Bending of nanowires using an atomic force microscope (AFM) is one of the most popular testing techniques for nanomaterial characterization. Wong et al. [1] performed AFM bending tests to directly measure the force-displacement relation and determined the mechanical properties of cantilever SiC beams by using conventional beam theory. Jing et al. [3] determined the elastic modulus of silver nanowires with diameters ranging from 20 to 140 nm by performing three-point bending tests on suspended nanowires. They found that the apparent Young's modulus of silver nanowires increased significantly with decreasing wire diameter.

Nanobeams are also key components of nanomechanical and nanoelectromechanical systems (NEMS) which are essentially sensors, actuators, machines and electronics at the nanoscale [4]. These devices can be used to measure extremely small displacements and forces that can lead to novel applications in engineering, advanced materials, medicine, computers, communications, etc. Current material processing technology allows for fabrication of NEMS of a few nanometers. The static and dynamic behavior of nanobeams is the principal feature that is exploited in the functional design of NEMS. Beams encountered in microelectromechanical systems (MEMS) are normally analyzed using the classical beam theory as classical theories are applicable at the microscale. However, it has been reported that at the nanoscale the response of beams is size-dependent and the conventional beam theory need to be modified [1-3].

To understand and predict the behavior of nanoscale structures, various modeling approaches have been proposed including atomistic simulation methods. The reason for the size-dependent behaviour at nanoscale is that the fraction of energy stored in surfaces becomes comparable with that in bulk due to the relatively high ratio of surface area to volume of nanoscale structures. Excess energy associated with surface/interface atoms is called surface/interfacial free energy. The ratio of surface free energy $\gamma\,(J/m^2)$ and Young's modulus $E\,(J/m^3)$, γ/E, is dimensional (m) and points to some other inherent parameter of a material [5]. This intrinsic length scale is usually small, in the nanometer range or even smaller. When a material element has one characteristic length comparable to the intrinsic scale, the surface/interface free energy can play an important role in its properties and behaviour. A direct method for analysis of nanoscale structures is to apply atomistic simulations but prohibitive computing cost makes it impractical.

This study is motivated by the need to develop a suitable mathematical model to understand the complex size-dependent behavior of nanobeams observed in experiments [1-3, 6, 7] and the need for a simple simulation tool to analyze beams in NEMS and other nanoscale devices. The classical beam theory widely used to analyze nanobeams does not account for important effects at the nanoscale such as surface energy. Gurtin and Murdoch [8, 9] presented a mathematical model that incorporates the effects of surface and interfacial energy into continuum mechanics.

A mechanistic model based on the Gurtin-Murdoch theory is first presented to analyze thin and thick nanoscale beams with an arbitrary cross-section. The main contribution of the first part of this study are a set of analytical solutions for static response of thin and thick beams under different loading (point and uniformly distributed loading) and boundary conditions (simply-supported, cantilevered and both ends fixed), and the solution of free vibration characteristics of such beams. Complete details of the analytical solution including the formulation of a new beam theory are given elsewhere [10] and its finite element formulation can be found in Ref. [11]. In the second part of this

study, a model for large deflections of thin beams is developed based on the Gurtin-Murdoch continuum theory and applied to examine the experimental results of Nilsson et al [7] and the classical large deflection beam model used by Søndergaard et al. [12]. The formulation of non-linear beam theory and solution algorithm is discussed. It is shown for the first time that good agreement with experiments can be obtained by using size-independent mechanical properties such as the bulk elastic modulus and surface residual stress. The model is then applied to show the influence of end boundary conditions and surface residual stress and resulting softening/stiffening effects. The present study shows that solutions obtained from the classical beam theory require careful interpretation when applied to nanobeams and generalization of nanomaterial behavior on the basis of classical beam models could lead to questionable conclusions.

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