THEORETICAL PREDICTION OF AGE DEPENDENT CRACK WIDTHS IN R/C BRIDGE GIRDERS FOR SUSTAINABLE ASSET MANAGEMENT

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Abstract

This paper discusses the viability of using theoretical crack prediction models as a means of identifying the most appropriate age for maintenance intervention. Usually bridge asset owners rely heavily on superficial observations, especially in early ages, 20-30 years, of a bridge. Visual inspection, a qualitative and subjective approach, is predominantly used as a prime method for bridge condition monitoring by the bridge asset owners. Desired outcomes of qualitative assessment have the limitations of providing satisfactory results and lead to failure in function and safety. Reinforced concrete is a live material which cracks due to shrinkage, creep and flexural stresses. Shrinkage cracks are dominant over the first few years, whereas the flexural and creep cracking, resulting from sustained and periodic loading, can continue whole of life. Depths and widths of these cracks, when reaching a critical level, encourage elements of corrosion at play. The system may enter a vicious-cycle of stiffness and strength degradation, without timely intervention. This paper investigates and proposes a methodology in theoretically predicting most appropriate time of intervention. Authors propose this methodology as a supplement to traditional practice of visual inspection for more informed decision making. It equips bridge owners with greater insight to what cannot be physically perceived by the human eye and could help decision-making to be more objective and cost effective. Research work presented in this paper is, in application, more relevant in reviewing existing bridge maintenance processes adopted by local, regional, state or national levels of governance. Methodology is demonstrated using time-series behaviour of a rectangular R/C section simulating a bridge girder. Post corrosion time series behaviour is not included within the scope of this discussion.

Keywords: Age Dependent, Crack Widths, Bridges, Asset Owners.

1. Introduction

In most of the developed metropolitans road bridges are constantly aging. According to Stewart (2001), in Australia over 60% of bridges for local roads are over 50 years old and approximately 55% of all highway bridges are over 20 years old. Count of in-service bridges, aging over 50 years was estimated to increase rapidly after 2011 and rise to 47.3% by 2021, in Kinki region of Japan (Ito et al., 2003). As bridges grow old they tend to decay quickly. Deterioration takes place due to many reasons, for example, increment in usage; in combination with the increase in magnitude of the imposed loading conditions, fluctuations within the surrounding environment and also due to variation in strains for example shrinkage, creep and elastic strains. It was estimated that US$300 million were urgently needed to strengthen or replace defective bridges in the Australian state of New South Wales (NSW) (Stewart, 2001). In the US 125,000 bridges were rated as structurally deficient (bridges that are restricted to light vehicles only, are closed or require immediate rehabilitation to remain open) – it was estimated that at least US$90 million was needed to rectify the problems (Stewart, 2001). Such incidences highlight the significance of age-dependent reliability analysis models for reinforced concrete bridges.

Condition monitoring of road infrastructures can be broadly categorised into two sections. Namely, the condition monitoring of pavements and the condition monitoring of road structures such as bridges, culverts, underpasses, overpasses, causeways. This paper limits its scope to road structures category, especially to bridge girders. According to Wu (2003) the widely used inspection method to assess the structural condition of bridges is visual inspection, on a regular basis. Also as per VicRoads (2011), one of the national road authorities in Australia, visual inspection is generally carried out at every 6 months to check the general serviceability of the structure, particularly for the safety of road users and to identify any emerging problems. It is mostly conducted with an objective of collecting all condition data to a component level required for the managements of road authorities (Casey, 2011).

Condition data is mostly recorded in state of words such as, for damage “minor”, “serious”, or “advanced”, for condition “good” or “bad” and for recommendations, “repair required” and “need further inspection”. These conditions are logged in condition rating sheets using the standard condition rating criteria, refer to (VicRoads, 2011). Sometimes, an inspector does not have clear access to get underneath the bridge or site due to high water level, extra weed grown underneath or around the bridge deck, deep valley, unavailability of appropriate machinery, such as, long boom cranes or bucket truck. As a result, inspector makes comments by just looking at the structural element, far from the actual position and occasionally without even been able to look at the structural element. These subjective outcomes are then used by asset owners to make decisions. Such decisions are not considered very reliable and thoughtful to society. As per a study conducted by Graybeal et al. (2001), in-depth inspections of portions of highway bridges were performed by 49 experienced state bridge inspectors, did not provide accurate and reliable inspection results. The delamination survey of a concrete bridge deck showed that the accuracy of this type of inspection was also relatively poor. Author (Graybeal et al., 2001) also suggested that such inspections can indicate the presence of large, widespread deficiencies such as corrosion and section loss, but can hardly locate the presence of crack indications that exist.

This paper objectively discusses the importance of quantitative assessment. Moreover, it proposes a methodology to supplement the traditional practise of visual inspection for more informed decision making and providing a structured pathway for the state's road asset owners, such as, councils, shires and state authorities. Theoretical prediction of critical crack widths will be demonstrated with the help of rectangular R/C bridge girder for ease of illustration.

2. Cracking

Time dependent cracking of R/C, as a composite material is a complex phenomenon and the theoretical prediction of it, has always been, the foremost concern for designers and researchers. Cracking occurs due to shrinkage, creep and flexural stresses which come with complexities in which various uncertainties stem from inherent material variations as well as from modelling unreliability (Yang, 2007).

Reinforced concrete beams, including pre-stressed and post tensioned beams, are widely used in bridge decks. They are the main structural elements subjected to flexural stresses under sustained imposed dead-loads and periodic traffic-loads throughout their life span. These critical structural elements have larger surface area and exposed to environmental elements triggering corrosion. Figure 1, illustrates the typical cracking patterns of a reinforced concrete beam, subjected to concentrated loads at 1/3 points, demonstrating, both, flexural and shear cracks. In real life situation these cracks are compounded by the shrinkage and creep induced cracks. It is considered that when a crack width reaches a critical value 0.3mm, the corrosion can sets in quite rapidly, depending on the environment within which it functions. In addition, well before such critical crack widths being reached from a strength limit state stand point, for a concrete beam to be serviceable, it is also necessary to control the deflection criteria where effective stiffness, and therefore the cracking, shall not reach a critical value resulting in excessive deflection (Gilbert, 2001).

Figure 1: A beam tested in lab showing crack patterns

3. Effect of Shrinkage and Creep on Cracking

Shrinkage, ε_{sh} of concrete is the time-dependent strain measured in an unloaded specimen at constant temperature (Gilbert, 2001). It can be classified into four categories plastic shrinkage, drying shrinkage, autogeneous shrinkage and carbonation shrinkage. All the categories allied with drying process involves the evaporation of absorbed water from the capillary pores of the cement paste (Warner et al., 1998) and reduces the volume of concrete. Furthermore, it is considered independent of the stresses applied and increases with time. Shrinkage in concrete thus produces compressive strain which also results in some downward deflection and instigates minor cracks.

Creep, on the other hand, is the increase in strain with time due to sustained load at a decreasing rate. It is classified in two forms, elastic and creep strain. Initial deformation due to load is the elastic strain, while the additional strain or time-dependent deformation due to the sustained load is creep strain. Creep is time dependent as well as stress dependent. In structural reliability analysis, time dependent problems are generally considered to be those in which the loading is modelled as time-variant and the resistance of the structure changes with time and/or loading (Li and Melchers, 1992). According to Gilbert (2011) creep is usually calculated as the difference between the total time-dependent deformation of a loaded specimen and the shrinkage of similar unloaded specimen. The capacity of concrete to creep is usually measured in terms of the creep coefficient, Φ (t, τ) Gilbert (2011). As a result of continuous loading, cracks appear at a higher rate on tensile surface of concrete.

It can be said that shrinkage and creep have significant impact on concrete structures (Al-Manaseer and Lam, 2005). They cause deflections and initiate cracking. Effects of shrinkage and creep are related to safety against failure and economic factors such as durability, serviceability, and long-term reliability (Al-Manaseer and Lam, 2005). While knowledge of material behaviour and other technical factors is critical, occasionally the decision maker's inaction has also contributed to structural failure. For example, I 35 W bridge (a steel structure) collapsed in Minneapolis, US, and the De La Concorde overpass in Montreal (a concrete structure), Canada. De La Concorde overpass failed due to horizontal cracking and rebar's improper detailing. Therefore, it is essential to establish the crack widths of bridge girders, at the early stage of life span.

4. Predictive Methods of Crack Widths

Predicting crack widths of in-service reinforced concrete beams have been a challenging task for engineers and researchers since 1960s. Various design tools and concepts have been invented to quantify the problem over the time (Allam et al., 2012). Several formulas are invented and proposed to forecast the crack widths (Chowdhury and Loo, 2001) and crack patterns. A number of researchers predicted the crack widths based on theoretical models and experimentation (Allam et al., 2012). An overview of such models and experimental work is discussed in Allam et al. (2012) and have been briefly mentioned here.

Saliger and Tomas used Bond-Slip model, Borms and Base et al. used No-Slip model, however Welch and Janjua and Leonhardt used Localized Bond Slip model to predict the crack width (Allam et al., 2012). Chowdhury and Loo (2001) proposed a new formula for prediction of crack widths based on test results which includes crack spacing and crack width measurements from 18 reinforced and 12 partially prestressed concrete beams. (Gilbert and Nejadi), tested 6 beams and 6 one-way slabs with different flexural reinforcement ratio and bar arrangement including various concrete cover. Makhlouf and Malhas (1996) also examined the consequences of thick concrete cover on the maximum flexural crack width under service load. Experiments by Broms and others have too showed that both crack spacing and crack width are related to the concrete cover distance, measured from the centre of the bar to the face of the concrete (Frosch, 1999). Based on these methods and experimental results, an allowable crack widths are suggested by codes and researchers,(Mosley et al., 2007), (Gilbert and Ranzi, 2011) and ACI Committee (2001).

4.1 Critical Crack Widths

The maximum critical crack width recommended by EuroCode2 is 0.3 mm for all exposure classes under the action of quasi-permanent combination of loads (Mosley et al., 2007). Whereas, according to Gilbert and Ranzi (2011), the maximum crack width for Australian conditions varies between 0.3 mm and 0.7 mm. However, American Concrete Institute Building code ACI Committee (2001), suggests that maximum crack width ranges between 0.10 mm to 0.41 mm, depending on the exposure conditions. For comparisons please refer to the tables 1, 2 and 3.

Exposure Class	**Reinforced members and prestressed members without bonded tendons**	**Prestressed members with bonded tendons**
	Quasi-permanent load combination	Frequent load combination
XO, XC1	0.3[a]	0.2
XC2, XC3, XC4	0.3	0.2[b]
XD1, XD2, XD3, XS1, XS2, XS3		Decompression
a) For XO, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed. b) For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads		

Table 1:- Eurocode 2: Allowable crack width limits. Ref (For exposure class definitions refer to Section4: MS EN 1992-1-1: 2010- table 1)

Environment	**Design Requirement**	**Maximum final crack width, w* (mm)**
Sheltered environment (where crack widths will not adversely affect durability	Aesthetic requirement	
	• where cracking could adversely affect the appearance of the structure • close in buildings • distant in buildings	 0.3 0.5
	• where cracking will not be visible and aesthetics is not important	0.7
Environment	Durability requirement	0.3
	• where wide cracks could lead to corrosion of reinforcement	
Aggressive environment	Durability requirement	0.30 (where c* ≥ 50 mm) 0.25 (otherwise)
	• where wide cracks could lead to corrosion of reinforcement	

Table 2:- Gilbert, 2011 Allowable crack width limits

Tolerable Crack Widths in Reinforced Concrete Structures, ACI 224		
	Exposure Condition	**Tolerable Crack Width, mm**
1.	Dry air or protective membrane	0.41
2.	Humidity, moist air, soil	0.30
3.	De-icing chemicals	0.18
4.	Seawater and seawater spray, wetting & drying	0.15
5.	Water-retaining structures (excluding non-pressure pipes)	0.10

Table 3:- ACI 224: Allowable crack width limits

4.2 Predictive method used

Model proposed by Gilbert (2011) is deployed in this paper to calculate the critical crack widths. In order to perform the analysis, a singly reinforced rectangular concrete beam is considered under serviceability bending moment of 58 KN.m. Figure 2 shows the layout of the R/C beam.

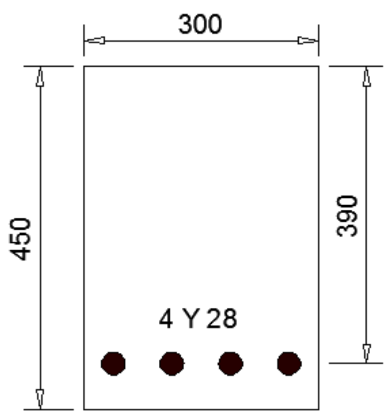

Figure 2: Cross section view of R/C rectangular beam

Gilbert (2011) provides the following equation for predicting the crack widths of flexural members

$$(w^*)_{soffit} = \frac{k_{cover}\, s^*}{E_s}\left[\frac{T}{A_{st}} - \frac{\tau_b s^*}{d_b}(1 + n_e * \rho_{tc}) - (\varepsilon_{sh} * E_s)\right] \quad (1)$$

Where,

$(w^*)_{soffit}$ = final maximum crack width at the member soffit, k_{cover} = term to account for the dependence of crack width on the clear concrete cover, c and varies with time due to change in d_n, neutral axis and can be calculated from the following equation

$$k_{cover} = \left(\frac{D-d_n}{d-d_n}\right) * \left(\frac{5c}{(D-d_n)-2d_b}\right)^{0.3} \quad (2)$$

s^* = final maximum crack spacing and is equal to $(0.67 * s_{max})$, s_{max} can be calculated from

$$s_{max} = \frac{f_{ct*} d_b}{2 * \tau_b * \rho_{tc}} \tag{3}$$

f_{ct} = flexural tensile strength, is defined as the maximum stress that the concrete can withstand when subjected to uniaxial tension. In the proposed model f_{ct} is replaced by $f'_{ct.f}$, the flexural tensile strength and may be estimated using Clause 3.1.13 of (AS3600, 2009). Whereas, d_b = bar diameter as shown in figure 1, τ_b = bond shear stress between the steel and the surrounding tensile concrete and ρ_{tc} = reinforcement ratio of the tension chord and may be taken as

$$\rho_{tc} = \frac{A_{st}}{A_{ct}} \tag{4}$$

$$A_{ct} = 0.5 * (D - d_n) b^* \tag{5}$$

b^* is the width of the section at the level of the centroid of the tensile steel.

$n_e = \frac{E_s}{E_e}$, the effective modular ratio,

$$E_e = \frac{E_c}{(1 + \Phi\ (t, \tau))} \tag{6}$$

E_c = elastic modulus of concrete and $\Phi\ (t, \tau)$ = creep coefficient. Both parameters are time dependent and can be calculated by using Clause 3.1.2 and 3.1.8 of (AS3600, 2009), respectively. E_s = elastic modulus of steel, taken as 200 GPa.

T = resultant tensile force estimated using A_{st} = area of steel, M_s = in service bending moment and I_{cr} = second moment of area of the fully-cracked section. ε_{sh} is shrinkage strain which may be computed using Clause 3.1.7 of (AS3600, 2009).

All the computations were performed in MS Excel by using above stated equations. Besides these equations, few additional equations (eqn 7 and 8) were used in order to perform the analysis.

$$I_{ef} = I_{cr} + (I_{uncr} - I_{cr}) * \left(\frac{M_{cr}}{M_s^*}\right)^3 \tag{7}$$

$$M_{cr} = Z\ (f'_{ct.f} - \sigma_{cs}) \tag{8}$$

Where Z is the section modulus of the uncracked section, referred to the extreme fibre at which cracking occurs and σ_{cs} is the maximum shrinkage-induced tensile stress. Few calculations have been performed separately and not discussed here due to scope limitation.

Most of the parameters in above mentioned equations are time dependent, such as, d_n, A_{ct}, ρ_{tc}, ε_{sh}, E_c, $\Phi\ (t, \tau)$, n_e, k_{cover}, τ_b , s^* , where, few decreases with time and some increases. For example, s^* decreases with time due to increase in number of cracks, τ_b decreases with time, probably as a result of shrinkage-induced slip and tensile creep (Gilbert and Ranzi, 2011). Creep coefficient, $\Phi\ (t, \tau)$, is one of the factors which increases with time because of regular stress increment, shrinkage strain ε_{sh} also rises alongside as a result of increment in drying and endogenous shrinkage. Whereas, model suggests that $(w^*)_{soffit}$, increases with time due to regular increment and decrement in subsequent parameters. Results obtained from the model has been discussed below and compared with study conducted by other researchers (Allam et al., 2012).

5. Results

To assess the accuracy of above mentioned equations for predicting crack width, a comparison is carried out with a study conducted by Allam et al. (2012). This study verifies the accuracy of building codes and equations developed by researchers, a comparison against some experimental data made available in the literature. Beam used in experimental study was designed by using 16 mm diameter reinforcements with 30mm cover and 370 mm depth. On the other hand, beam used for theoretical analysis is shown in figure 2. Theoretically predicted crack widths for R/C rectangular beam are plotted against tensile steel stress, figure 3 and later compared with experimental outputs, figure 4. Comparison is done against tensile steel stress because it is one of the most important factors that affect crack width and is directly proportional to bond shear stress; τ_b. Tensile steel stress increases due to increment in tension force, reduction in concrete resistivity and cover loss.

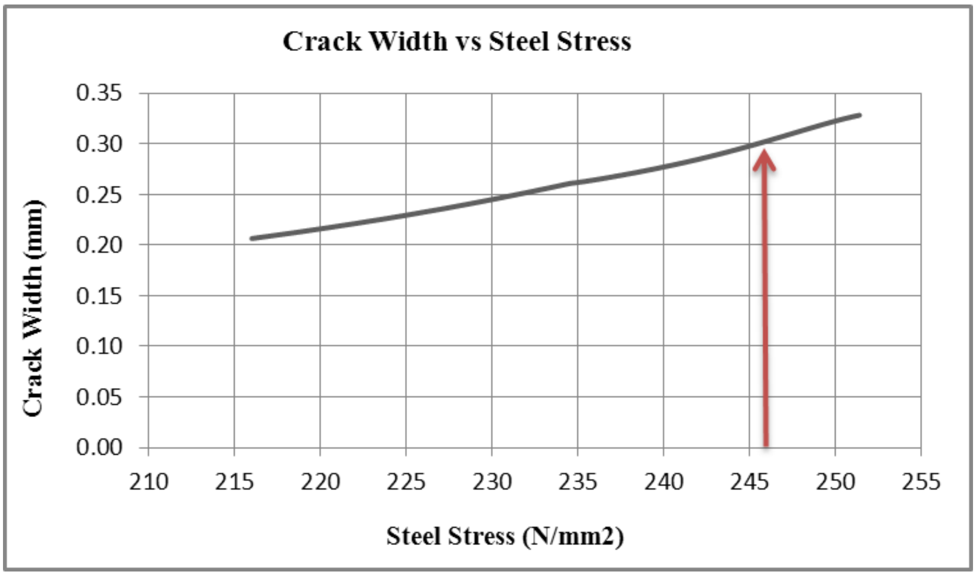

Figure 3: Theoretically predicted crack widths drawn against tensile steel stress

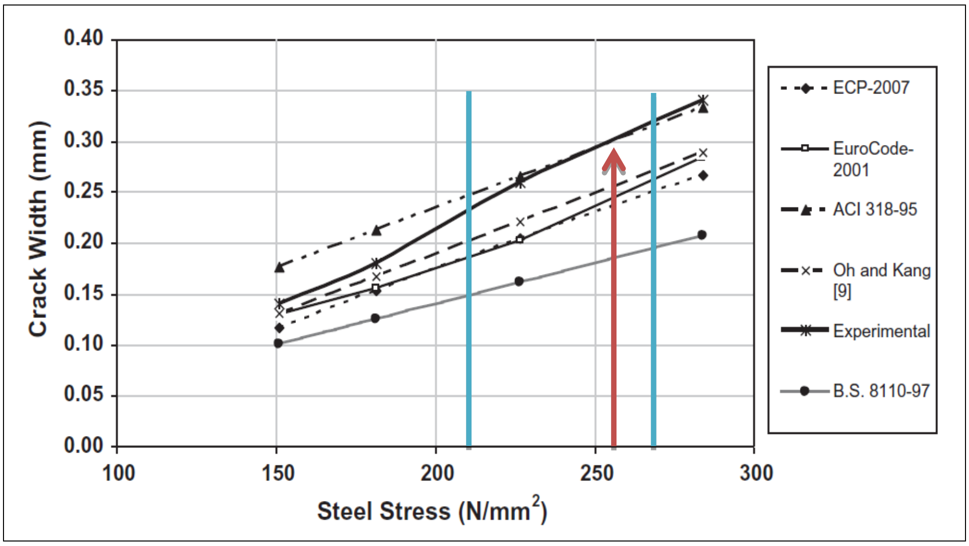

Figure 4: Crack widths predicted by several researchers (Allam et al., 2012)

Figure 4 contains a diverse tensile steel stress zone, but values indicated between 210 to 270 MPa (blue lines zone) are only compared with figure 3 results. This is due to several unknown input parameters, such as, compressive strength of concrete, loading type, shear stress used in figure 4 analysis.

Comparing the results of above graphs, it can be indicated that first crack width, i.e. 0.21mm in figure 3 appears at 216 MPa (N/mm2), whereas, in figure 4 it ranges between 0.10mm to 0.16mm at 150 MPa. However, values commencing from 216 MPa in figure 4 are only compared and discussed here. So, primary crack width at 216 MPa in figure 4 is 0.23mm, which is very close to the theoretically predicted value, that is, 0.21mm. Hence the difference between theoretical prediction and experimental crack width is 0.02mm. This difference can be a result of, different compressive strengths of concrete, various tensile steel, fault in handling the material and also as a result of the quantity, orientation and distribution of the reinforcing steel crossing the cracks (Gilbert, 2008). Variability in concrete cover also influences the irregularity between crack width and patterns.

As per the guidelines and codes (ACI 224, AS 3600-2009 and Eurocode2), 0.3mm is considered as allowable crack width. Figure 3 achieves this value at 246 MPa, whereas, figure 4 showed a large scatter among the different codes and guidelines and between all it was attained by ECP-2007 and experimental work at 252 MPa. Reason for achieving the critical value at different steel stress levels can be due to difficulty in determining bond strength and variable steel grade. This comparison is quite evident to say that experimental value counterparts theoretical output and validates the analysis.

6. Research Application

In future, this research will focus on the viability of theoretical predictive models in recognising the most appropriate and opportune time for maintenance intervention in critical structural elements of bridge assets. The key objective is in providing deterministic, theoretical underpinning to visually inspected condition data. It is envisaged that such a reciprocal mechanism, by which visually inspected data – of structural significance – can be supplant by deterministic theoretical evaluations. It is expected such a tool would equip the asset owners in informed decision making.

Designing of this tool has commenced with first phase (evaluation of the crack widths with time) been virtually accomplished. Figure 5 shows the output of the finished work for an R/C rectangular beam.

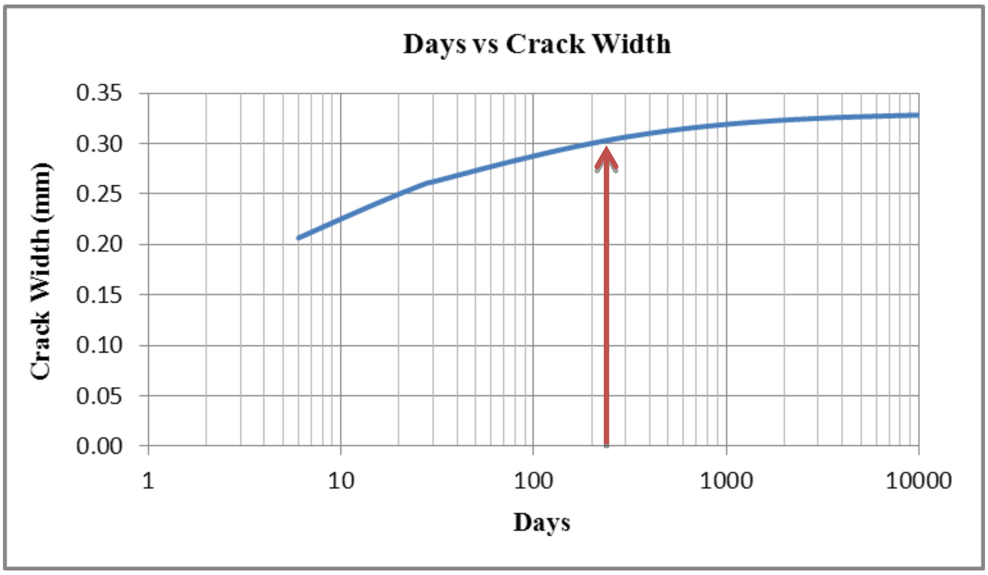

Figure 5: Theoretically predicted crack widths versus Time

Figure 5 indicates that the beam reaches the maximum crack width limit, i.e. 0.3 mm in 1 year of time. This early cracking can be because of shrinkage and creep. It theoretically triggers asset owners and provides an appropriate time of intervention and inspects the bridge, rather than heavily relying on superficial observations. Crack width is increasing from 0.30mm to 0.32mm over the span of 28 years and expected to rise further due to low concrete ductility and its constant degradation over time.

Critical crack widths and depths mentioned above increase the porosity and allow elements of corrosion to set in motion. Therefore, developing a relation between critical widths and their impact on corrosion pattern will also be demonstrated in later part of the research. Values drawn in figure 5 are still not validated and will be certified as soon as Level 2 inspection data, comprising of, age of the structure, basic measurements, present condition rating is made available to the research team.

At present, basic parameters such as width, length, depth, and hypothetical thickness, compressive strength of concrete, area of steel and concrete are used as inputs to perform the analysis. Outputs are documented by using the elementary response parameters of plain and reinforced concrete, such as elastic modulus, creep, shrinkage and crack width. Additional input parameters such as location of structure, as built design conditions, age of the structure, and type of load applied, number of spans will be added to the existing model in order to improve the age dependent prediction.

7. Discussion

- Deterioration takes place due to many reasons, for example, increment in usage; in combination with the increase in magnitude of the imposed loading conditions, fluctuations within the surrounding environment and also due to variation in strains for example shrinkage, creep and elastic strains.

- In beams, cracking usually takes place due to shrinkage and creep. They have significant impact on concrete structures and cause deflections and initiate cracking.

- An overview of crack width prediction models and experimental work was discussed in comparison with theoretical model.

- The maximum critical crack width value recommended by EuroCode2, AS3600-2009 and ACI is 0.3mm.

- Model proposed by Gilbert (2011) was deployed in the paper to calculate the critical crack widths. After that, theoretically predicted values were validated against experimental work conducted by other researchers.

- Some of the time-dependent parameters were discussed and how those parameters vary with time (increase or decrease) was highlighted. These basic parameters were incorporated in the crack width calculations.

- An age-dependent graph was plotted showing the crack width behaviour for an R/C rectangular beam over 30 years of life span. From the above graph it was established that the beam reaches the maximum limit, i.e. 0.3 mm in 1 year and theoretically triggers asset owners to inspect the bridge. Still this graph is not validated and will be proven once level 2 inspection data becomes available.

- The key objective of this research is in providing deterministic, theoretical underpinning to visually inspected condition data and providing a tool to asset owners which would assist them in informed decision making.

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