PREDICTION OF COMPRESSIVE STRENGTH OF CONCRETE

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ABSTRACT

This report presents a method of predicting the compressive strength of concrete. The approach is based on experimental investigation of compressive strength development of four mix proportions with three different brands of cement. The criterion used for the prediction of strength involves the determination of equivalent age of concrete, which depends on curing temperature and age of concrete. It was found that the compressive strength of concrete varies linearly with respect to the logarithmic value of the equivalent age. Furthermore, it was observed that the rate of change of strength with respect to logarithmic value of equivalent age varies non-linearly with respect to w/c ratio. This variation was not the same for all cement brands tested. Expressions were derived for compressive strength for each brand of cement with respect to w/c ratio, age and curing temperature. Based on these expressions, a chart was developed to establish the relationship between strength and w/c ratio for a range of strength grades. This chart can be used in mix design of concrete using local materials. Since it was not possible to develop a unique expression for the prediction of compressive strength of concrete in terms of mix design parameters such as w/c for any cement brand, the identified behaviour of the strength development of concrete with respect to equivalent age was used in predicting strength of concrete. In the proposed method, the 28-day strength can be predicted with strength results at two ages such as 1-day and 3-day strengths. Good agreement with the predicted and experimental results have been obtained not only for test results obtained under this study but also for other published test data.

INTRODUCTION

The strength developed by concrete made with given materials and given proportions increases for many months under favourable conditions, but in the majority of specifications the strength is specified at the age of 28 days. It is a common practice to continue construction during this 28-day period and by the time the 28-day strength is known, a considerable amount of construction may have been carried out. If the 28-day strength of concrete does not meet the specified strength then the remedial actions will be extremely difficult and costly due to progress of construction work. In such situations, prediction of compressive strength of concrete, specially the 28-day strength is very important to detect first sign of change in the 28-day strength and to make necessary changes to the mix proportion to restore the expected 28-day strength. Early prediction of the 28-day strength can save lot of money if concrete strength does not meet the required strength. On the other hand, prediction of concrete strength will also help the contractor to make decisions such as when the concrete is strong enough for the removal of formwork, when slabs can be used without damage, and when the construction load can be applied. Furthermore, it would be useful for quality control purposes at ready-mixed concrete plants.

Many researchers have investigated methods of predicting concrete strength using various testing procedures and conducting laboratory experiments [1,2,3,4]. Some investigators used properties of fresh concrete to predict early age strength based on statistical analysis [2]. In addition, several attempts have been made to relate concrete strength to its age and curing temperature [3,4].

The main objective of this research is to predict the 28-day compressive strength of concrete in terms of early age strength of concrete, which is easily measurable in the field.

EXPERIMENTAL INVESTIGATION

Based on the literature survey[1,2,3,4], it was decided to investigate the strength development of concrete for a period of 28 days for different grades of concrete under different curing temperatures. Also the effect of source of cement (brand of cement) on strength development of concrete was also investigated.

Concrete Mixes

The physical properties of aggregates were determined in accordance with relevant BS specifications[5] and given in Table 1. Concrete mixes for four different grades (i.e. Grade 20, 25, 30, and 50) were designed based on DoE method[6]. Table 2 gives the mix proportion for all mixes. Three OPC cement brands available in the local market were selected for this study. This selection was based on the fact that these brands have been widely used in the local construction industry.

Table 1 Physical properties of aggregates

Aggregate	Property		
	Relative Density (SSD)	Bulk density (kg/m^3)	Water absorption (%)
Fine	2.65	1636	0.52
Coarse	2.76	1608	0.19

Table 2 Mix Proportions (per $1m^3$)

Mix No.	Grade	Cement (kg)	Water (kg)	Fine Aggregate (kg)	Coarse Aggregate (kg)	W/C
1	20	282.33	181.03	834.53	1176.67	0.64
2	25	316.67	183.64	747.00	1129.40	0.58
3	30	380.00	183.67	731.42	1176.12	0.48
4	50	423.70	184.28	675.71	1165.70	0.43

Testing Procedure

Twelve test cubes of 150mm× 150mm× 150 mm were cast from each mix proportion and for each cement brand to obtain compressive strength at the ages 1 day, 3 days, 7 days and 28 days. The slump and the compacting factor were determined for each batch in accordance with BS specifications [7]. This procedure was repeated for all three cement brands and the four mix proportions. Thus a total of 144 cubes were cast.

The cubes were tested in accordance with BS 1881: Part 4: 1970 after curing under laboratory conditions for the required period. The temperature of the curing tank was recorded daily.

ANALYSIS OF RESULTS

Test results

The results of the Cube Tests are given in Table 3. The slump was always around 25 mm while the compacting factor was almost 0.85 for all the batches. The minimum and the maximum values of the ambient temperature in the curing tank were found to be 26 ^{0}C and 28 ^{0}C respectively during the entire period.

Equivalent Age concept

The strength development of concrete made with all types of Portland cement depends on the temperature and humidity condition during curing. Concrete gains strength more rapidly at higher temperatures due to the increase in speed of the chemical reaction. There are two terms, namely Maturity and Equivalent age, to express a relationship between strength, time and temperature so that the strength of a particular concrete after any particular time and temperature cycle can be established from the knowledge of its strength after any other time and temperature cycle.

Maturity is the age of a particular concrete expressed as degree-hour, i.e. as the area under a temperature-time curve.

The Equivalent Age is the age at which a particular concrete would attain its current strength and degree of hydration, if maintained at a nominated standard temperature. It can be expressed by the following equation [8].

$$EA = \sum \left(te^{-q\left(\frac{1}{T_a}-\frac{1}{T_S}\right)} \right) \qquad (1)$$

Where,
EA = Equivalent Age *(hours)*
q = Activation Energy / Universal Gas Constant
T_a = Temperature (actual) for the time interval 't' (K)
T_s = Standard (reference) temperature *(K)*
t = Time spent at temperature 'T_a' *(hours)*

Table 2 Summary of test results

Brand Of Cement		"CSN"		"CME"		"CRH"	
Mix No.	Age (days)	Average* Ambient Temp. (°C)	Average Cube Strength (N/mm^2)	Average* Ambient Temp. (°C)	Average Cube Strength (N/mm^2)	Average* Ambient Temp. (°C)	Average Cube Strength (N/mm^2)
1	1	26.00	5.17	27.50	6.98	27.00	7.35
	3	26.00	9.36	-	-	27.25	12.17
	4	-	-	27.33	16.32	-	-
	7	26.50	12.18	27.00	18.37	27.33	15.54
	28	27.33	18.54	27.28	28.07	27.14	21.82
2	1	28.00	6.30	26.50	7.44	27.00	9.02
	3	28.00	10.16	26.88	15.21	27.50	14.64
	7	27.83	14.34	26.92	21.06	27.42	19.62
	28	27.40	23.20	27.22	30.11	27.14	25.86
3	1	28.00	7.73	26.75	11.23	28.00	12.57
	3	27.75	12.54	26.75	23.43	-	-
	4	-	-	-	-	27.67	22.67
	7	27.83	17.09	26.75	31.26	27.58	25.83
	28	27.45	28.43	27.25	42.18	27.14	34.30
4	1	28.00	9.59	27.25	16.67	27.25	12.70
	3	28.00	17.07	-	-	27.38	23.24
	4	-	-	27.17	34.26	-	-
	7	-	-	27.25	39.46	27.58	28.40
	8	27.75	24.92	-	-	-	-
	28	27.38	38.42	27.26	53.05	27.01	38.11

* This denotes the average daily ambient temperature in the curing tank during a particular period of curing.

In this study, T_a denotes the average ambient temperature of the curing medium. For local conditions, T_s is taken as 27°C (300K). Generally, the value of 'q' is 4200.

Since the equivalent age concept is more accurate [9], it was used to analyze the strength data.

Compressive strength development

Figures 1,2 & 3 show the variation of strength with Equivalent Age (EA) for "CSN", "CME" and "CRH" cement respectively. It can be seen that the variation of strength with *Log(EA)* is almost linear for all three brands of cement and for all four mix proportions.

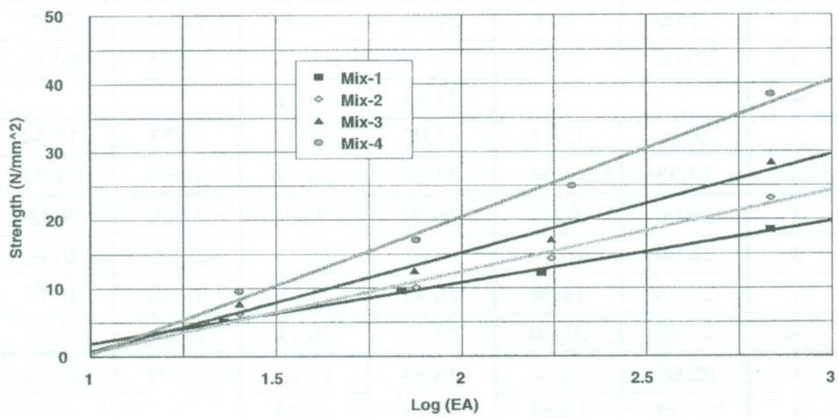

Figure 1 Variation of Strength with Log(EA) for "CSN"

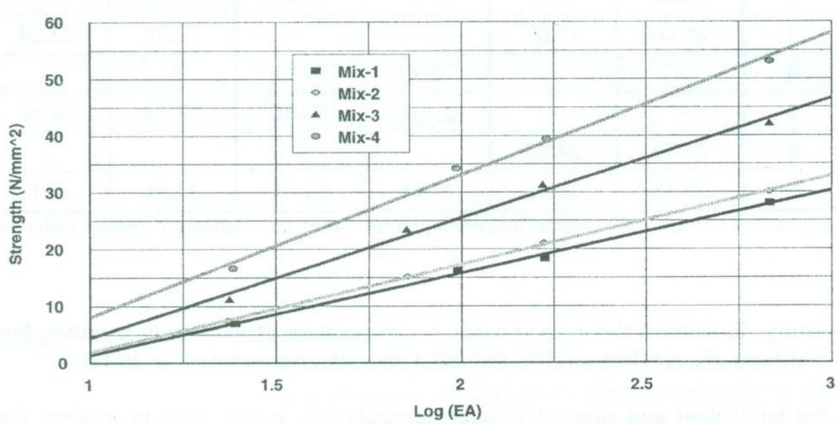

Figure 2 Variation of Strength with Log(EA) for "CME" cement

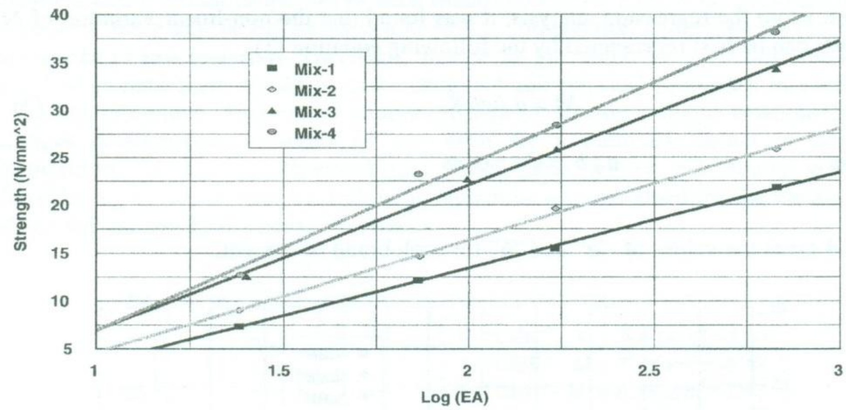

Figure 3 Variation of Strength with Log(EA) for "CRH"

Thus, for a particular concrete mix, the strength 'F_{cu}' at age 't' can be expressed as follows.

$$(F_{cu})_t = M \ Log(EA)_t + C \qquad (2)$$

Where, M = Gradient of " Strength vs. Log(EA) "
C = Y - intercept

Table 3 gives the summary of regression curves for each mix shown in Figures 1,2 & 3.

Table 3 Summary of regression curves in Figures 1, 2 &3

Mix	w/c	Cement brand								
		"CSN"			"CME"			"CRH"		
		M	C	R^*	M	C	R^*	M	C	R^*
1	0.64	9.02	-7.29	0.99	14.50	-13.13	0.99	9.93	-6.39	0.999
2	0.58	11.83	-11.22	0.98	15.52	-13.64	0.99	11.73	-7.08	0.997
3	0.48	14.38	-13.58	0.98	21.20	-16.86	0.99	15.14	-8.22	0.997
4	0.43	19.98	-19.56	0.99	24.97	-16.69	0.99	17.20	-10.13	0.991

* *Coefficient of correlation*

Figure 4 shows the variation of '*M*' with the water-cement ratio for each brand of cement. From the regression analysis, it was found that the non-linear variation of ***M*** with ***w/c*** can be best represented by the following equation (3).

$$M = a\,(w/c)^{b} \qquad \textbf{(3)}$$

Where, a , b = Constants

Table 4 gives the values of "*a*" and "*b*" for each brand of cement.

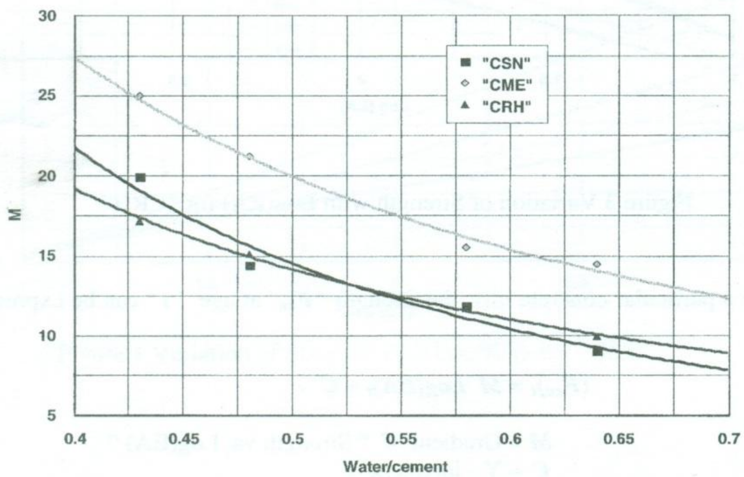

Figure 4 Variation of ***M*** with ***w/c***

Table 4 Power Regression Data of " M vs. W/C "

Brand of Cement	Regression curve data		
	a	*b*	Coefficient of correlation R
"CSN"	4.11	-1.82	0.958
"CME"	7.46	-1.42	0.986
"CRH"	5.46	-1.37	0.996

It is clear from Figure 5, that the strength development is different for the same mix proportion with different brands of cement. This can be due to the difference in chemical and physical properties of each brand of cement. Even with the same brand of cement, there can be differences in chemical composition depending on the batch and source of raw materials. Therefore, there is no guarantee that even with the same brand of cement there would be same chemical and physical properties in each and every bag of cement. This makes it more difficult to develop a unique relationship for strength development in terms of w/c even for a particular brand of cement. Furthermore, it can be observed that these three curves are nearly parallel. Considering this behaviour, the following equations were obtained for the variation of ***M*** and ***w/c*** for the three brands of cements.

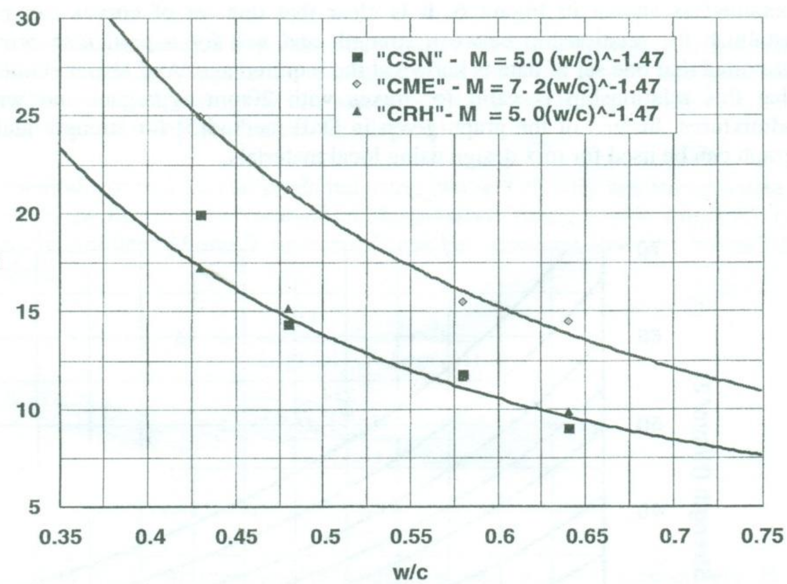

Figure 5 ***M*** vs. ***w/c*** for all brands of cement

For "CSN" Cement - $M = 5.0(w/c)^{-1.47}$ (4.a)

For "CME" Cement - $M = 7.2\ (w/c)^{-1.47}$ (4.b)

For "CRH" Cement - $M = 5.0\ (w/c)^{-1.47}$ (4.c)

Table 6 Summary of ***C/M***

Mix	w/c	Brand of cement					
		"CSN"		"CME"		"CRH"	
		C/M	Average	C/M	Average	*C/M*	Average
1	0.64	0.808	0.919	0.905	0.812	0.643	0.594
2	0.58	0.948		0.879		0.603	
3	0.48	0.944		0.795		0.543	
4	0.43	0.978		0.668		0.588	

In addition, it can be seen from Table 6, that the C/M ratio for three brands of cement is approximately constant for each brand of cement. Therefore, the following equations can be derived for the strength of concrete in terms of w/c for the three brands of cement.

$$\text{For "CSN"} \quad (F_{cu})_t = 5.0(w/c)^{-1.47}\left(\log(EA)_t - 0.92\right) \quad (5.a)$$

$$\text{For "CME"} \quad (F_{cu})_t = 7.2(w/c)^{-1.47}\left(\log(EA)_t - 0.81\right) \quad (5.b)$$

$$\text{For "CRH"} \quad (F_{cu})_t = 5.0(w/c)^{-1.47}\left(\log(EA)_t - 0.59\right) \quad (5.c)$$

Based on the above equations, the strength variation with ***w/c*** for different ages can be obtained as shown in Figure 6. It is clear that this set of curves can be used to establish the relationship between strength and w/c for a particular cement brand provided that one set of data is known at the required age. And also it should be noted that this relationship is valid for mixes with 20mm aggregate and without any admixtures. Instead of the graph given in DoE method[5] for strength and w/c, this graph can be used for mix design using local materials.

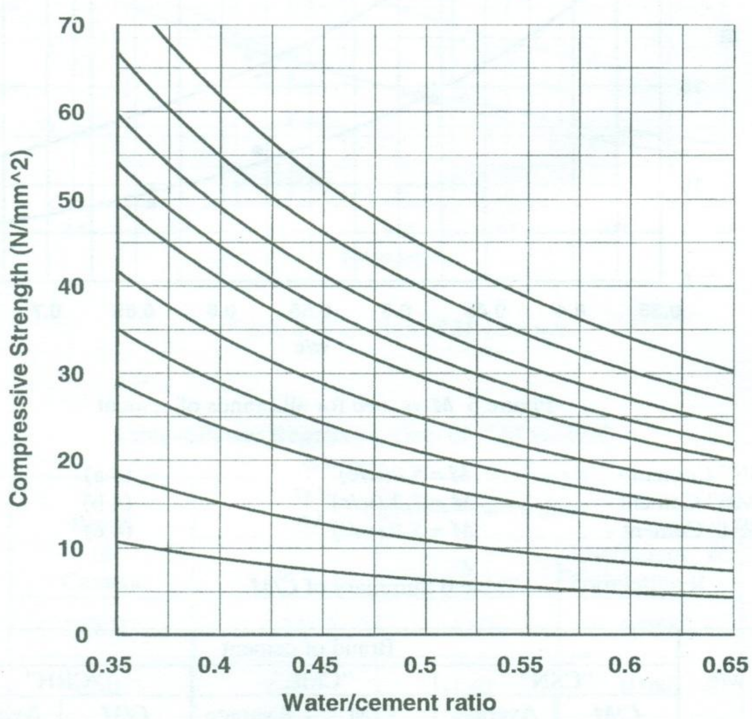

Figure 6 Relationship between compressive strength and w/c ratio

The above three equations (5.a),(5.b) & (5.c) can be written in the general form as follows.

$$(F_{cu})_t = m(w/c)^{-1.47}\left(\log(EA)_t - k\right)$$

Taking the average value of k for three brands of cement, compressive strength of concrete can be approximately represented by the following equation.

$$(F_{cu})_t = m(w/c)^{-1.47}\left(\log(EA)_t - 0.77\right)$$

The above equation can be used to predict strength of concrete if the w/c ratio and one set of strength data at known age are available. Therefore the 28-day compressive strength can be predicted with early age strength data. If the mix proportion is not available, then the following method can be adopted.

Under this method, strength can be predicted using two sets of early age strength data. This is based on the fact that the variation of compressive strength with Log(EA) is linear. Thus, the gradient ***M*** and Y intercept ***C*** can be calculated from the following two expressions.

$$M = \frac{(F_{cu})_{t1} - (F_{cu})_{t2}}{Log(EA)_{t1} - Log(EA)_{t2}}$$

$$C = \frac{Log(EA)_{t1}(F_{cu})_{t2} - Log(EA)_{t2}(F_{cu})_{t1}}{Log(EA)_{t1} - Log(EA)_{t2}}$$

Where $(F_{cu})_{t1}$ and $(F_{cu})_{t2}$ are compressive strength at ages t_1 and t_2 respectively. By substituting this ***M*** and ***C*** values in Equation (2), strength can be predicted up to 28 days irrespective of brand of cement and w/c ratio.

PREDICTION OF COMPRESSIVE STRENGTH

Prediction was done for the 28 day compressive strength of each mix (i.e. t=28 days). The earliest age at which the strength data were available was used as ' t_0' . Also, the Equivalent Age for 28 days ($(EA)_{28}$) was calculated based on the average ambient temperature for the period ' t_0'. Tables 7 gives the predicted 28-day strength for strength data given in Table 2. Since the real test of any relation is its ability to fit data other than data from which it has been derived, the proposed relation was applied to data from other sources. Table 8 shows the predicted 28-day strength based on 3-day and 7-day strength data from other sources which shows the proposed equation correlates the test data with a significant accuracy. It should be noted that, when the ambient temperature was not available, it was assumed to be same as the reference temperature.

Table 7 Prediction of 28-day Strength using 3-day and 7-day strengths given in Table 2

Brand of cement	W/c	Actual strength (N/mm^2)	Predicted strength* (N/mm^2)	Variation (%)	Predicted strength** (N/mm^2)	Variation (%)
"CSN"	0.64	18.54	17.88	3.55	16.79	9.4
	0.58	23.20	18.01	22.3	21.18	8.7
	0.48	28.43	22.32	21.5	24.53	13.7
	0.43	38.42	32.28	16.0	34.95	9.0
"CME"	0.64	28.07	29.43	4.8	23.45	16.4
	0.58	30.11	31.01	3.0	30.63	1.7
	0.48	42.18	48.23	14.3	44.33	5.1
	0.43	53.05	58.95	11.1	52.34	1.3
"CRH"	0.64	21.82	21.97	0.7	21.05	3.5
	0.58	25.86	26.07	0.8	27.77	7.4
	0.48	34.30	36.85	7.4	33.66	1.9
	0.43	38.11	44.67	17.2	36.84	3.3
Average (Std. dev.)				10.22 (7.89)		6.80 (4.87)

* *Predicted strength using 1day and 3 days (or 4 days) strength data*
** *Predicted strength using 3 days and 7 days (or 8 days) strength data*

Table 8 Predicted 28-day Strength for test data from other sources

Source	Measured strength (N/mm^2)		28-day strength (N/mm^2)		% error
	3 days	7 days	Measured	Predicted	
[5]	27.0	36.0	49.0	50.7	+3.47
[10]	24.5	31.6	39.4	43.1	+9.39
	35.5	40.6	49.7	48.8	-1.81
	21.5	26.7	35.9	35.1	-2.22
[11]	24.8	33.9	46.1	48.8	+5.85
Average					4.55

CONCLUSIONS

According to the results, 60% to 75% of the 28-day strength of concrete is achieved in the first 7 days. It was found that the variation of compressive strength varies linearly with respect to the logarithmic value of Equivalent age of concrete. Furthermore, it was found that the gradient of " Strength vs. Log(EA) " is a function of the water-cement ratio. In addition, it was found that the strength development was different for the same mix proportion with different cement brands. Three equations were derived for prediction of strength in terms of w/c ratio and equivalent age for the three brands of cement used. Based on these equations, a series of curves were developed for the relationship between strength and water-cement ratio. These sets of curves can be used in mix design of concrete using local materials.

The following general form of the relationship between strength and equivalent age was proposed to predict the 28-day strength from two sets of early age strength data.

$$(F_{cu})_t = M \, Log(EA)_t + C$$

Thus, the gradient (***M***) and Y intercept ***C*** can be calculated from the following two expressions.

$$C = \frac{Log(EA)_{t1}(F_{cu})_{t2} - Log(EA)_{t2}(F_{cu})_{t1}}{Log(EA)_{t1} - Log(EA)_{t2}}$$

$$M = \frac{(F_{cu})_{t1} - (F_{cu})_{t2}}{Log(EA)_{t1} - Log(EA)_{t2}}$$

Where $(F_{cu})_{t1}$ and $(F_{cu})_{t2}$ are compressive strengths at ages t_1 and t_2 respectively. It was found that with 3-day and 7-day strength data, the 28-day strength could be predicted with a significant accuracy.

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