# DEVELOPMENT AND EVALUATION OF THE AESTHETICS OF STRUCTURAL FORM

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#### **ABSTRACT**

This Paper demonstrates that the aesthetic concepts of engineering design do not just arise; but that they are derived from various models of aesthetics. It also presents various Proportioning Systems and their application in Structures, through case studies, notably the use of the Golden Proportion.

The Paper then describes the investigation of the aesthetic perception of two sets of respondents and also the optimisation of a simple structure. The preference for equality and for the Golden Ratio is discovered from the experiments on aesthetic perception. The study also deals with the optimisation of simple structure.

Finally, it also describes the possibility of making initial design decisions relating to dimensions, using the plots of optimisation and plots of aesthetic preferences.

#### 1. INTRODUCTION

The main purpose of designing a structure is to satisfy certain functional needs. However, designing a structure involves many considerations, in addition to the functional requirements. They are, topography, site forces, character of the place, technical requirements, etc. Therefore aesthetics of a structure is an outcome of the synthesis of all these considerations with the creativity of the designer.

Whatever the structure is, it can only be realised if the total cost of the structure falls within the allocated budget. The cost of a structure depends on many factors. Among them 'scale and proportion' is one of the most influential factors, which is also linked with aesthetics as well. Therefore, it would be useful to study the variation of cost with respect to scale and proportion of a particular structure.

If the designer has the above information, then, he would be able to design structures with the required aesthetic qualities within the allocated budget. This study attempts to find the above information.

'Engineering' and 'Aesthetics' are not pure, definite or isolated disciplines. They have links with many other disciplines as well. Therefore, a study related to 'Engineering and Aesthetics' can not be done in isolation. Hence, this study is also integrated with Art, Architecture, Philosophy and Sociology.

However the scope is limited to the following objectives.

#### 2. OBJECTIVES

The main objectives of this study are:

- To study basic aesthetic concepts and proportioning systems used in design.
- 2. To examine the application of proportioning systems in structures.
- To investigate the perception of proportions and of the perception of aesthetics of simple structures.
- 4. To optimise a simple structure, using simple measures.
- 5. To identify ranges of dimensions that a designer should work with, in order to be within the allocated budget.

#### 3. METHODOLOGY

In order to achieve the above objectives, the following methodologies were adopted.

- 1. A literature review was carried out to determine the basic aesthetic concepts and proportioning systems used in design.
- Case studies were used to examine the application of proportioning systems in structures.
- 3. Social surveys were carried out to investigate the perception of proportions and perception of the aesthetics of a three span bridge.
- A simplified approach was used to find the optimisation curve for a given structure.
- The optimisation curve was used to identify the ranges of dimensions that a designer should work with, in order to be within the allocated budget.

#### 4. AESTHETICS IN DESIGN

The word 'aesthetics' had been derived from the Greek word 'aisthesis', meaning sensory perception. It was regarded as a branch of philosophy concerned with the understanding of beauty and its manifestation in art and nature. However it is extremely difficult to give a precise definition for aesthetics.

Aesthetic consciousness covers special sentiments, tastes, interests, concepts, ideals, views and theories<sup>2</sup>. Therefore, for any aesthetically conscious designer, it is useful to study the theories of aesthetics.

However, adherence to any such theory may reduce creativity. Hence, what is important is not the blind application of any such theory, but the intelligent and intuitive understanding of aesthetic concepts and their application in design creatively.

#### 4.1 Models of Aesthetics

Aesthetic concepts do not just arise. They are derived from various models of aesthetics. Therefore for any designer, it is important to be aware of the various models of aesthetics, which have been developed through the centuries.

In the course of its development, aesthetics has evolved five major theoretical models.<sup>3</sup>

## Model 1 (Objective Idealism):

The aesthetic appears when God or the Ideal spiritualizes the world.

## Model 2 (Subjective Idealism):

The aesthetic appears when the individual's inner wealth is superimposed on life, which is aesthetically neutral.

#### Model 3 (Dualism):

The aesthetic is produced by a union of the objective and the subjective.

## Model 4 (Metaphysical Naturalism):

Regards aesthetic characteristics as the natural properties of objects like, say weight symmetric composition, colour or shape.

## Model 5 (Dialectical Materialism):

Treats the aesthetic as an objective property of phenomena and objects which is a result of their relations with the life of society and mankind.

## 4.2 Aesthetic Concepts in Structural Design

As mentioned above, aesthetic concepts are derived from various aesthetic models. Some basic aesthetic concepts are given below.

- "Structures should harmonise with nature" this is derived from the model of metaphysical naturalism.
  e.g.- Clustered water tanks at Alencon, France, look like trees in nature. (Figure 1a)
- "Structures need not necessarily harmonise with nature, the conflict between the structure and the environment may create new progressive aesthetic values"— this is derived from the model of dialectical materialism.
  e.g.- Eiffel Tower in Paris<sup>5</sup> contradicted with the environment and the contemporary society, when it was erected. However, later it was able
  - e.g.- Eiffel Tower in Paris<sup>3</sup> contradicted with the environment and the contemporary society, when it was erected. However, later it was able to create new aesthetic values and become the Semiotic Symbol of France. (Figure 1b)
- 3. "Structures should subjectively be related with the consumer or the user" this is derived from the model of subjective idealism.

e.g.- Water Tower at Fisons Fertilizer Factory, UK, 6 is subjectively related with the factory workers and expresses the feeling of sharing the work. (Figure 1c)

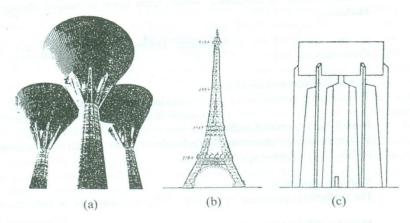


Fig. 1 – Aesthetic Concepts and their application in Structures 4, 5, 6

In addition to the above-mentioned historical aesthetic models, new models can be created. Based on these new models, new aesthetic concepts too can be derived.

Aesthetic concepts in design are always linked with the visual aspect. It is not sufficient that the object or structure functions satisfactorily; its appearance must be visually satisfactory. Indeed, appearance may be an essential part of purpose or use.<sup>4</sup>

Appearance of any object is linked with scale, size, volume, proportion, styling, light and shade, optical illusions, etc. Among these, 'Proportion' is one of the major factors that influence the appearance of an object. Therefore this study concentrates on aesthetic beauty and proportions.

## 4.3 Aesthetic Beauty and Proportions

Proportions play an essential part in aesthetic beauty. Certain arrangements in the proportion of shape and form result in pleasurable sensations. Lack of it leads to a reaction of indifference, discomfort or even revulsion.

The early Greek philosophers tried to define aesthetic beauty through the geometrical laws of proportion. They believed that beauty is harmony and that harmony is due to the observation of proportions.

Throughout history, artifacts and buildings, which have been universally recognised as good examples of aesthetic beauty, have used proportion as the foundation of their designs.<sup>8</sup>

## 4.4 Proportioning Systems

A number of proportional theories have been put forward in the search for aesthetic beauty, each gaining favour from time to time.

From the ancients, we have two mathematical systems that can bring a sense of proportion to the world of design.<sup>9</sup>

- 1. Commensurable (or rational) system of proportion.
- 2. Incommensurable (or irrational) system of proportion.

## 4.4.1 Commensurable System of Proportions

Since the Renaissance, and perhaps since Greek and Roman times, history seems to have favoured the commensurable system of proportions and its simple ratios like 1:2, 2:3 and 3:4. In music these numbers are connected to what we hear and understand to be harmony and consonance. The ratios 1:2, 2:3 and 3:4 in the harmonic series are related with the major scale of western music. The ratio 1:2 of the harmonic series is designated as the 8<sup>th</sup> or the *octave*. Similarly ratios 2:3 and 3:4 are designated as *fifth* and *fourth* respectively. The somewhat hollow-sounding 4<sup>th</sup>, 5<sup>th</sup> and 8<sup>th</sup> of the major scale are called *Perfect Intervals*. They possess what we may perhaps call a 'purity' distinguishing them from other intervals. When spaces or objects are organised and designed around these simple whole-number ratios, they convey a sense of musical harmony.

## 4.4.2 Incommensurable System of Proportions

While the Vitruvian Canon reflects a desire for musical harmony that is very much in keeping with the human anatomy, another ancient system of proportions offered a different, perhaps even richer, method of finding harmony between the parts and the whole. This is done not through wholenumber ratios that produce the harmonics in music, but through incommensurable ratios, like 1:  $\sqrt{2}$ , 1:  $\sqrt{3}$ , and 1:  $\phi$  (or the Golden Mean). These are the very same numbers that appear in simple geometric shapes, such as the square, the triangle and the pentagon. (Figure 2)

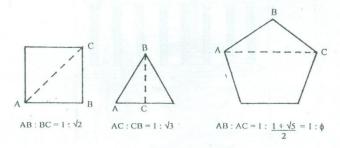


Fig. 2 – Incommensurable ratios of simple geometric shapes

Of all the incommensurable proportions, the most elegant and efficient way to achieve harmony is the Golden Mean. The Golden Mean Proportion appears in nature constantly, from sunflowers, apple blossoms, and daisies in the plant world to spiral shells beneath the seas. The spirals of pinecones exhibit this proportion, as do artichokes, pussy willows, and pine apple husks. Many of the proportions of the human body also conform to the Golden Mean. The Golden Ratio is found when a line is divided into two unequal lengths so that the shorter relates to the longer as the longer relates to the whole. The ratio is identified by the Greek letter  $\phi$ , and it translates numerically to a ratio of 1: (1 +  $\sqrt{5}$ ) / 2 or 1: 1.618.

#### 5. APPLICATION OF GOLDEN PROPORTION IN STRUCTURES

Throughout the history and prehistory of architecture, the Golden Mean has consistently brought a sense of harmony to design, not to mention a spiritual and even a physical sense of well-being. A number of examples could be cited, including Stonehenge and the Great Pyramids in Egypt; classical temples of Greece; Gothic Cathedrals; and modern buildings. Some of these structures are given below.

## 5.1 Parthenon in Athens, Greece

Even from the time of the early Greeks, a rectangle whose sides are in the "Golden Proportion" has been known. This rectangle is supposed to appear in many of the proportions of that famous ancient Greek temple, the Parthenon, in Athens, Greece, built around 400 BC. <sup>10</sup>(Figure 3)

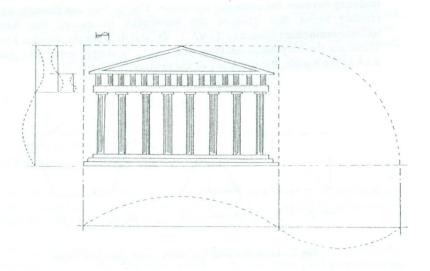


Fig. 3 - Golden Proportions of Parthenon, Greece.

## 5.2 Cistercian Abbey, Fontenay

The Golden proportion can also be identified in nature and be developed from basic forms and shapes of spatial order, to music and even to a sense of time. Through this development, the Abbeys of St. Bernard and Le Corbusier's La Tourette Monastry achieved not only simplicity and visual beauty but also through the reverberation of sound within the church, they transformed human chanting and singing into supposedly celestial music. (Figure 4)

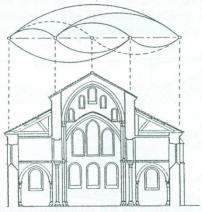


Fig. 4 - Golden Proportions of Cistercian Abbey, Fontenay 7

## 5.3 Yakushiji Temple

The use of the golden proportion is not exclusive to Western culture for there is evidence of its use in both the art and architecture of the East. In Japan, for example, the Pagoda of the Yakushiji Temple, which is known for its soaring grace and ingenious structural strength employs the golden section to define the heights and dimensions of its six roofs. (Figure 5)

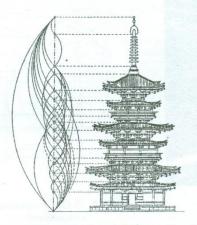


Fig. 5 - Golden Proportions of Yakushiji Temple Pagoda, Japan<sup>7</sup>

## 5.4 Redheugh bridge in Newcastle

Redheugh Bridge in Newcastle is a Three-span bridge. The Golden Mean has been employed to decide the lengths of the three spans. 8 (Figure 6)

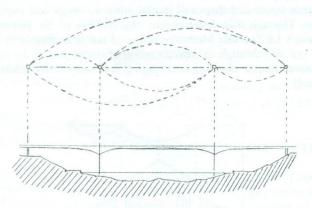


Fig. 6 - Golden Proportions of Redheugh bridge<sup>8</sup>

## 5.5 Ting Kau Bridge, Hong Kong

The arrangement conceived for the cable-stayed solution of the Ting Kau Bridge in Hong Kong involved a twin span triple towered pylon. The central tower, while it mirrored the outer two, had to be structurally much stiffer and therefore became the focus for the composition. In order to make the whole composition visually attractive the golden proportion was fully integrated into the design. (Figures 7 and 8)



Fig. 7 Ting Kau Bridge<sup>7</sup>

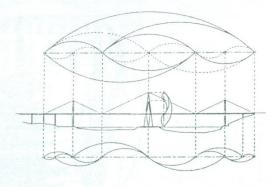


Fig. 8- Golden Proportions of the Ting Kau Bridge composition<sup>7</sup>

## 5.6 Vasco da Gama bridge, Portugal

The pylons for the second Tagus road crossing in Portugal are conceived as pure sculpted elements, as if carved from a single piece. All the surfaces flow smoothly one into another with uncomplicated simple lines. The pylon's legs are cranked to allow vertical cable profiles and a sense of the concentration of the forces is expressed by the subtle taper in the legs below the change in angle. The omissions of a cross beam immediately below the deck at the pylons visually articulates the fact that the deck is suspended throughout. The tie beam, required to withstand lateral and seismic forces, is carefully positioned according to golden proportions and completes the composition. (Figures 9 and 10)

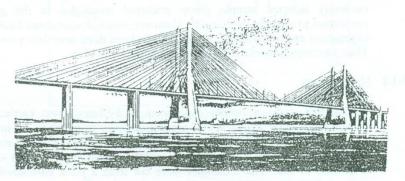


Fig. 9- Twin-towered Vasco da Gama bridges.8

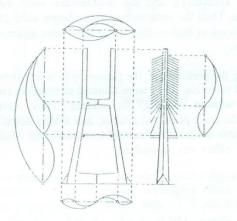


Fig. 10 - Golden Proportions of the Pylon of the Vasco da Gama Bridge.<sup>8</sup>

#### 6 EVALUATION OF AESTHETICS

## 6.1 Perception of Proportions

## 6.1.1 Introduction

It is possible that some people are quite unaware of proportions in the physical aspects of things. Just as some people are colour blind, so some may be oblivious to the sensory stimuli of shape and form. However it is probably more reasonable to assume that people unaware of proportions are rare. It is more likely that they have an underdeveloped perception of proportions.

Georg Th. Fechner, in his laboratory experiments, studied the aesthetic preferences of common people with no aesthetic training. Fechner's experiments in 1876 were extensive, showing that over 75% of a large, randomly selected sample group preferred rectangles in the golden proportion to any other rectangle<sup>8</sup>. However, much details about Fechner's experiment are not available. Further it has been done over 100 years ago. Therefore repeating of his experiment can be worthwhile.

## 6.1.2 Methodology

During this study, a similar experiment to Fechner's was carried out. Eleven rectangular shapes whose proportions vary from 1.0 to 2.0 in steps of 0.1 were used. Here one unit was taken as 40 mm. Thus the smallest rectangle was 40 mm x 40 mm in size and the largest rectangle was 40 mm x 80 mm in size. The rectangles were cut from a thick cardboard. The colour of the rectangle was white. These eleven rectangles were kept randomly on a large brown colour board and the respondents were asked to select their preferences.

The rectangular shapes were first placed horizontally (i.e. shorter dimension vertically), and the respondents were asked to select their preferences. Then the same rectangular shapes were placed vertically (i.e. shorter dimension horizontally), and they were again asked to select their preferences.

In selecting the preferences, the respondents were allowed to use their own methods of selection. They were allowed to translate (move horizontally or vertically without rotating) the shapes. They were also allowed to eliminate the least preferred shapes, and finally select the most preferred shape. By this method the 'central tendency' was avoided and the 'lost in the shapes' effect was minimised.

## 6.1.3 Results

In the above study 100 randomly selected people were interviewed. Only four of them expressed their inability to identify the most preferred shape.

One had an idea that any shape would be pleasing. The other three had a view that there was no significant perceptual difference between the shapes. However 96 respondents out of 100 were able to select their preferences. The summary of results is shown in Figures 11 and 12. (where  $\alpha = \text{longer dimension of the rectangle}$ )

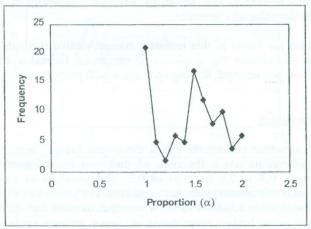


Fig. 11 - Preferences when shorter dimension is vertical

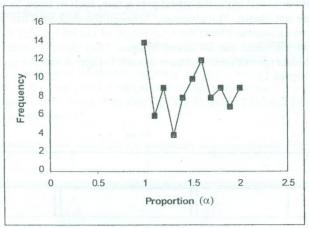


Fig. 12 - Preferences when shorter dimension is horizontal

The results show that most of the respondents prefer squares. After that a considerable number of people prefer rectangles close to the Golden Rectangle, with proportions on either side of it showing diminishing preferences.

## 6.2 Perception of Aesthetics of Structures

## 6.2.1 Introduction

In the true sense, the aesthetic value of a structure depends on many factors such as scale, size, proportion, hollow and volume, colour, light and shade, styling, optical illusions, compatibility, integrity, rhythm, harmony, balance, unity, etc. In addition aesthetic taste is a highly subjective matter. This shows the extreme complexity of evaluating the aesthetic value of a structure.

Within the limits of this research, comprehensive methods could not be used to evaluate the aesthetics of structures. Instead a very simplified method was adopted, dealing once again with proportions.

## 6.2.2 Methodology

The structure considered was a three-span bridge. Seven bridges were considered in which the ratio of mid-span to end-span length varied between 0.8 to 2.0 in steps of 0.2. The positions of the two exterior supports (abutments) were kept constant. The positions of the two interior supports were decided in such a way that the two end spans were equal. These seven bridges were drawn on seven separate cards. The size of a card was 150 mm x 30 mm. The bridges were drawn in black on white background. Only the bridge deck together with a simple railing, abutments, piers and the water level was shown. Other surrounding objects that could influence the visual perception, such as trees, flow of water, etc. were not drawn. The clearance between the deck and the water level was taken as one-twelfth of the total length of the bridge. All the above factors were the same for all seven bridges. Only the positions of the interior supports (piers) were different in each bridge. A sample drawing is shown in Figure 13.

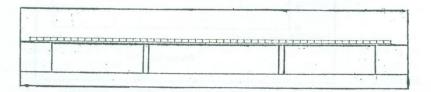


Fig. 13 - Sample drawing of a Three-span Bridge

These cards were kept randomly on the table and respondents were asked to select their preferences as explained in the previous experiment.

## 6.2.3 Results

As the above mentioned diagrams were technical in nature, only Civil Engineers were interviewed. In selecting preferences the respondents were advised to consider only the aesthetic appearance, but not the stability, economy, etc. The size of the sample was 50. The summary of the results is shown in Figure 14. (where  $\alpha = \text{length of mid span} / \text{length of end span})$ 

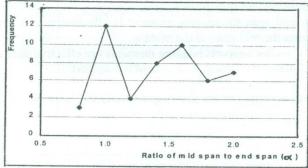


Fig.14 - Respondents' Preferences for the Three-span Bridge

The results show that most of the respondents prefer equal span continuous bridges. There is however another maximum around the golden section proportion.

## 7 OPTIMISATION OF SIMPLE STRUCTURES

## 7.1 Problem Description and Assumptions

For the purpose of analysis, a 30 unit long three-span continuous beam with a uniformly distributed load of 1 unit weight/unit length was considered. The positions of the two exterior supports were kept constant. The positions of the two interior supports were decided in such a way that the two end spans were equal. The shape of the Bending Moment Diagram is as shown in the Figure 15.

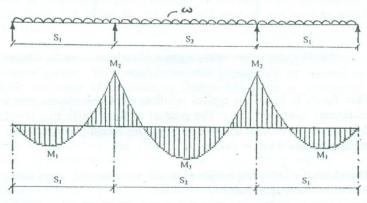


Fig.15 - Bending Moments in the Three-span Beam

The ratio of the mid-span to the end-span (i.e.  $\alpha = S_2/S_1$ ) was varied between 0.5 to 2.0 in steps of 0.1, and the whole family of beams were analysed.

For the purpose of optimisation a simple measure was defined. This measure was based on certain assumptions.

- In end spans the sagging moments are effective over a length of 0.8 times the end span from exterior support.
- 2. In mid span the sagging moment is effective over a length of 0.6 times the mid span, at the middle.
- 3. At interior supports hogging moments are effective over a length of 0.2 times the span at each side of the support.

The above assumptions are shown graphically in Figure 16.

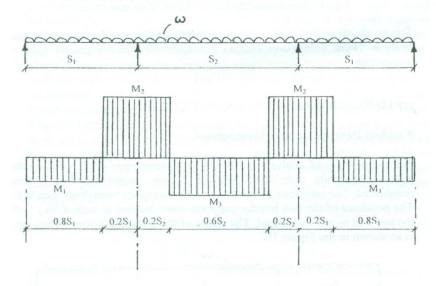


Fig. 16 - Lengths over which sagging and hogging moments are effective.

This figure is based on typical reinforcement curtailment practices for reinforced concrete beams. The product of these "effective lengths" and corresponding maximum moments will be roughly proportional to the material required for the bridge.

Therefore, the following simple measure was obtained, as an index for the cost of the three span beam.

$$f(M) = 2S_1 \{ 0.8 | M_1 | + 0.2 | M_2 | \} + S_2 \{ 0.6 | M_3 | + 0.4 | M_2 | \}$$

## 7.2 Results

The variation of the above measure with respect to  $\alpha$  is shown in Figure 17.(where  $\alpha = \text{length of mid span / length of end span)}$ 

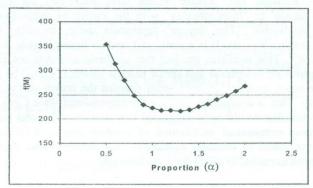


Fig. 17- Optimisation Curve for Three-span Beam

It shows a minimum (though not a very marked one) at a proportion around 1.4, a value close to  $\sqrt{2}$ .

#### 8. DISCUSSION

In the investigation on 'Perception of Rectangular Shapes' it was found that most respondents prefer rectangles of equal side (i.e. square shapes). After that a considerable number of respondents prefer rectangles close to the Golden Rectangle.

The second investigation was about the perception of span lengths of a 'three-span bridge'. Here it was found that most of the respondents prefer a three-span bridge having equal spans. Secondly, once again a reasonable number of respondents prefer bridges whose spans are decided by the Golden Ratio (i.e. when the ratio of mid-span to end-span is equal to Golden Ratio).

The results of the second investigation tally well with the results of the first investigation. This emphasises the importance of derivation of aesthetic concepts of engineering design based on a philosophy of proportions.

The desire for the Golden Rectangle or Ratio may be linked with an 'interest in nature', as the Golden Ratio appears in nature. Here the aesthetic appreciation might have been based mainly on the 'Visual Perception' or the 'Surface Sense' with the model of metaphysical naturalism (Section 4.1) being applicable. This desire for the Golden Ratio was evident in the structures shown in a previous section as well (Section

5). The ratio was evident in both facades and longitudinal views of both buildings and bridges, in both horizontal and vertical directions.

However, the most significant result is the desire for the Square. This cannot be explained merely by the 'Visual Perception' or the 'Surface Sense'. Perhaps, the 'Square' might have addressed the respondent's 'Inner Sense' or 'Intellectuality' with the model of subjective idealism being applicable. 'The Square' represents unity, equality, simplicity, purify, rationality, etc. It is a static and neutral figure having no preferred direction<sup>11</sup>. This confirms the fact that aesthetic concepts are linked with the various philosophical and social models (Section 4.1). The preference for the equal span bridge once again denotes the power of the concept of equality. This is despite the fact that the respondents were Civil Engineers whose training would have suggested that shorter end spans would result in a more economical distribution of bending moments (Although they were not asked to consider economy, their subconscious thoughts may have been expected to affect their preferences).

The optimisation curve has a minimum value close to  $\alpha = \sqrt{2}$ , i.e. when the ratio of mid-span to end-span is around  $\sqrt{2}$ . Although the structure selected was an ideal structure and only the simplest loading case (i.e. a uniformly distributed load) was considered for the purpose of analysis, there is nevertheless a well known value of  $\sqrt{2}$  that is obtained as the optimum proportion. This is not too far away from the Golden Ratio either. Hence there may be some correspondence between aesthetic sense and optimal solutions.

The plots of optimisation and plots of aesthetic preferences will enable designers to make initial design-decisions relating to dimensions (or proportions). If there are plateaus (or inverted plateaus), in both or either of the plots, then the designer would be able to work within a wide range without adversely affecting the economy.

## 9 CONCLUSION

This study leads to the following conclusions.

- 1. Aesthetic concepts in engineering design can be derived from the models of aesthetics.
- 2. The golden ratio was evident in the proportions of existing structures, both buildings and bridges, in both facades and longitudinal views, in both horizontal and vertical direction.
- 3. The experiments on perception indicated that the concept of equality was a very strong aesthetic principle, reflected in choices for the square (among other rectangles) and a three span bridge with equal spans (compared to other bridges of varying mid-span to end-span ratios).
- 4. The above experiments indicated that the Golden Ratio also received considerable support, whether in the form of a Golden Rectangle or in

the ratio of mid to end spans of a three span bridge being close to the Golden Ratio.

5. The preference for equality could be an application of the aesthetic model of subjective idealism, while that for the Golden Ratio an application of the aesthetic model of metaphysical naturalism.

6. The albeit simplified optimisation of a three span bridge yielded the optimum ratio of mid to end spans as being close to √2, another classical proportion similar to the Golden Ratio. The fact that this value is not too far away from the Golden Ratio suggests that there may be a correspondence between aesthetic sense and optimal solutions.

7. The use of optimisation plots (e.g. Figure 17) and plots of aesthetic preference (e.g. Figure 14) will enable designers to make initial design decisions relating to dimensions (or proportions).

## 10 ACKNOWLEDGEMENTS

We wish to thank all the respondents who participated in the experiments on perception.

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