Abstract: A numerical model based on two-dimensional nonlinear long-wave equations that include drag forces and turbulence induced shear force due to the presence of vegetation was developed for estimating tree breaking. The numerical model was then applied to a coastal forest, where two dominant tropical vegetation tree species were considered, *Pandanus odoratissimus* and *Casuarina equisetifolia*. Quantitative effects of the coastal forest destruction by tsunami on the decrement of tsunami force behind the forest were evaluated with or without including the destruction mode into the model. The analysis satisfies the previous field investigation knowledge that the critical breaking tsunami water depth is around 80% of the tree height when tree height is larger than 2m for *P. odoratissimus*. *P. odoratissimus* can reduce tsunami force higher than *C. equisetifolia* due to the complex of aerial root structures. Even when the trees are destructed at just above the aerial roots, tsunami force reduction rate are decreased by 20%, 10% for 2-4m trees, 6-8 m trees, respectively, because of the existence of dense aerial roots in case of a *P. odoratissimus*. The previous numerical models that do not include the breaking phenomena have a possibility to overestimate the vegetation effect for reducing tsunami force. The reduction of tsunami mitigation effect by breaking is larger for *C. equisetifolia*, however their growth rate is larger than *P. odoratissimus* and is hardly broken. The combination of *P. odoratissimus* and *C. equisetifolia* was recommended as a vegetation bioshield to protect coastal area from tsunami hazards.

Keywords: Vegetation bioshield, Trunk breakage, *Pandanus odoratissimus*, *Casuarina equisetifolia*, Tsunami force and moment acting on tree trunk

1 Introduction

Coastal vegetation has been found to play a significant role in reducing the tsunami energy and damage to humans and properties (Shuto, 1987; Danielsen et al. 2005; Tanaka et al., 2007), although their role is still questioned due to the absence of adequate studies (Kerr and Baird, 2007). Based on the field investigations carried out in Sri Lanka and Thailand after the Indian Ocean tsunami and in Indonesia after the Java tsunami on 2006, Tanaka et al. (2007) pointed out that *Pandanus odoratissimus* grown on beach sand is especially effective in providing protection from tsunami damage due to its density and complex aerial root structure, but it is not strong enough to withstand more than 5m-tsunami. The tsunami water depth for bending or breaking *P. odoratissimus* was observed above 80% of the tree height (Tanaka and Sasaki, 2007). On the other hand, *Casuarina equisetifolia*, one of the other dominant vegetation was found to be effective in providing protection from tsunami due to its higher density, especially they are young. *C. equisetifolia* does not have aerial roots and its trunks were rarely broken, however, young *C. equisetifolia* (less than 10cm-trunk diameter) were broken at the Indian Ocean tsunami (Tanaka et al., 2007, 2009). Observation reported that most of the damage to *C. equisetifolia* of which trunk diameter is larger than 15 cm was uprooting by scouring around roots. However, the uprooting was limited at the front region of forest against tsunami.

Some previous studies have discussed the effects of vegetation on tsunami mitigation based on the numerical simulation results (Nandasena et al. 2008, Tanaka et al. 2009, Thuy et al. 2009a and b, Thuy et al. 2010). However, the effect of tree breakage was not considered, because the tsunami water depth...
in their simulation range was less than 80% of the tree height.

Therefore the objective of this study is to know quantitatively how the breaking of trees in a forest affects on the tsunami disaster mitigation effects. In the present paper, a coastal forest of *P. odoratissimus* or *C. equisetifolia* was considered and potential tsunami force and bending moments on a tree were studied by numerical simulations. The numerical model is based on two-dimensional nonlinear long-wave equations as the same in Thuy et al. (2009a) which was developed for including tree breaking effect. The numerical results are validated with field measurement data of the threshold water depth for tree breaking, and then breaking length, reduction of water depth and potential tsunami force are discussed with the tree growth stage.

### 2 Mathematical model and calculation procedure

#### 2.1 Governing equations

The governing equations are two-dimensional nonlinear long-wave equations that include drag and eddy viscosity forces due to interaction with vegetation. The continuity and the momentum equations are respectively:

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial (dV_x)}{\partial x} + \frac{\partial (dV_y)}{\partial y} = 0
\]

\[
\frac{\partial V_x}{\partial t} + V_y \frac{\partial V_x}{\partial x} + V_x \frac{\partial V_y}{\partial y} + g \frac{\partial \zeta}{\partial x} + \frac{\tau_v}{\rho d} \frac{d}{d} - \frac{E_v}{d} = 0
\]

\[
\frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_x}{\partial y} + g \frac{\partial \zeta}{\partial y} + \frac{\tau_v}{\rho d} \frac{d}{d} + \frac{F}{d} = 0
\]

where,

\[
\tau_v = \frac{Pgn^2}{d^{1/3}} \left| \frac{\partial}{\partial z} \right| d
\]

\[
F = \gamma \frac{1}{2} \rho C_{D,all} b_{ref} \left| \frac{\partial}{\partial z} \right| d
\]

\[
E_x = 2 \frac{\partial}{\partial x} \left( dV_x \frac{\partial V_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( dV_x \frac{\partial V_y}{\partial y} + dV_y \frac{\partial V_x}{\partial x} \right)
\]

\[
E_y = 2 \frac{\partial}{\partial y} \left( dV_y \frac{\partial V_y}{\partial y} \right) + \frac{\partial}{\partial x} \left( dV_y \frac{\partial V_x}{\partial x} + dV_x \frac{\partial V_y}{\partial y} \right)
\]

\[
C_{D,all}(d) = C_{D,ref} \int b(z_G) \frac{C_D(z_G)}{C_{D,ref}} dz_G = C_{D,ref} \int \alpha(z_G) \beta(z_G) dz_G
\]

where \(b(z_G)\) and \(C_D(z_G)\) are the projected width and drag coefficient of a tree at the height \(z_G\) from the ground surface, and \(b_{ref}\) and \(C_{D,ref}\) are the reference projected width and reference drag coefficient of the trunk at \(z_G=1.2\) m in principle, respectively. The eddy viscosity coefficient \(\nu_e\) is expressed in the SDS turbulence model (Nadaoka and Yagi 1998, Thuy et al. 2009a). For details of \(\alpha(z_G)\), \(\beta(z_G)\), and \(C_{D,all}\) of *P. odoratissimus* and *C. equisetifolia*, see Tanaka et al. (2007) and Thuy et al.(2010).
2.2 Definitions of tsunami force and bending moment on a tree

The tsunami force vector ($\vec{F}^\rho$) in the present study is defined by the following equation:

$$\vec{F}^\rho = \frac{1}{2} \rho d \frac{\vec{V}}{\rho}$$

(9)

This is the potential tsunami force integrated over the inundation depth and corresponds to the total drag force due to the tsunami acting on a virtual tall column of unit width and a unit drag coefficient (Thuy et al., 2010).

On the other hand, the tsunami bending moment vector ($\vec{M}_{\text{tree}}$) at a critical height ($h_c$) of the tree from the ground surface is expressed as follows:

$$\vec{M}_{\text{tree}} = \frac{1}{2} \rho C_{D,\text{ref}} h_{\text{ref}} \frac{\vec{V}}{\rho} \int_{h_c}^{H_{\text{tree}}} \alpha \beta (z_G - h_c) d z_G, \quad h_c < d \leq H_{\text{tree}}$$

$$= 0, \quad d \leq h_c$$

(10)

where $h_c$ is critical height (top of aerial of for $P. odoratissimus$, =0 for $C. equisetifolia$). The notation of $\vec{M}_{\text{tree}}$ is replaced by $\vec{M}_P$ and $\vec{M}_C$ hereafter for $P. odoratissimus$ and $C. equisetifolia$.

Tanaka et al. (2009) proposed an empirical formula to calculate the breaking moment (unit: Nm) $M_{BP}$ and $M_{CP}$ of $P. odoratissimus$ and $C. equisetifolia$. The equation converted into SI unit is as follows:

$$M_{BP} = 4.45 \left(100 b_{\text{ref}}\right)^{2.62} \quad \text{for } P. odoratissimus$$

$$M_{CP} = 4.90 \left(1.5 \times 100 b_{\text{ref}}\right)^3 \quad \text{for } C. equisetifolia$$

(11)

where the empirical constants 4.45 and 4.90 have a dimension.

2.3 Method of numerical simulations, Coastal topography and forest conditions

A set of the above equations is solved by the finite-difference method of a staggered leap-frog scheme which is widely used in numerical simulations of tsunamis. The numerical scheme is described in detail in Thuy et al. (2009a). A sinusoidal incident tsunami was given as a time-dependent boundary condition at the most offshore side of the wave-generation zone. In the numerical simulation, the uniform grid size of 2.5 m was applied.

A uniform coastal topography with the cross-shore section perpendicular ($x$-axis) to a straight shoreline, as shown in Figure 1(a), was selected as a model case. The bed profile of the domain consists of four slopes, $S=1/10, 1/100, 1/50,$ and $1/500$. The offshore water depth at an additional wave-generation zone with a horizontal bottom is 100 m below the datum level of $z=0$. The tide level at the attack of the tsunami was considered to be 2 m, and therefore the still water level is 2 m above the datum level. The direction of the incident tsunami is perpendicular to the shoreline. In the present paper, the run-up of the first wave only is discussed. The coastal forest starts at the starting point of the 1/500 slope on the land ($x=5700$ m), where the ground is 4 m above the datum level (2 m above the tide level at the tsunami event). The forest was assumed to extend finite in the direction of the shoreline ($y$-axis). The value of $C_{D,\text{all}}$ varied with the total depth $d$ (inundation depth) because the projected width $b$ and the drag coefficient $C_D$ vary with the height from the ground surface $z_G$. Figure 1 (b) shows the variation of $C_{D,\text{all}}$ of $P. odoratissimus$ by water depth with tree growth stage (2, 4, 6 and 8 m tree height).

3 Results and discussion

3.1 Tsunami bending moment on a tree –Breaking of $P. odoratissimus$

Figure 2(a) and (b) shows the comparison of time profile of water depth and velocity in front and behind the forest for two models: N.B.M. and B.M.. The velocity and water depth behind the forest is increased after tree breaking owing to the reduction in drag resistance. However, in front of vegetation, the water depth decreases while the velocity increases due to the reduction of the reflection from vegetation.
Figure 3 shows spatial distribution of the maximum tsunami moments along the forest for two models. According to the result, the damage length of forest by B.M. is about 77.5 m (77.5%), and increased in comparison with N.B.M. due to the decrease in vegetation resistance in the front area where vegetation was broken.

Figure 4 shows the relationship between the reduction rate of water depth ($d_{\text{max}}/d_{\text{max0}}$), tsunami force ($F_{\text{max}}/F_{\text{max0}}$) and survival rate of $P.\ odoratissimus$ (number of unbroken trees /total tree) and incident tsunami water depth, where subscript 0 indicates the case of no forest. The results show that trees start breaking at $H_{F0}=4.8$ m. When $H_{F0}$ reaches to 5.5 m, all the trees are broken. The reduction of water depth and tsunami force is about 25 and 54 % respectively, and it decreases when the incident tsunami water depth exceeds the initial breaking of water depth (I.B. in the figure).

Figure 1: Condition of numerical simulation, (a) schematic of topography for numerical simulation, (b) variation of $C_{D-all}$ with water depth for four cases of $P.\ odoratissimus$ height

Figure 2: Time profile of water depth and velocity. (a) at front of forest, (b) at behind the coastal forest

Figure 3: Damage length simulated by two numerical models
3.2 Damage of P. odoratissimus by tsunami with tree growth stage - Validation the numerical results with field measurement data

The threshold water depths for tree breaking obtained from field measurement in Sri Lanka and Thailand after Indian Ocean tsunami in 2004 and Indonesia in Java tsunami 2006 were used to validate the numerical model. Breaking of vegetation depends on the tsunami height and the tree height. Figure 5(a) shows the observation of tree conditions (broken or not) against tsunami water depth for different tree height. As already discussed by Tanaka et al. (2009), most of the tree was broken when the tsunami water depth exceeds 80% of tree height. To validate the numerical model for the observed threshold values of water depth for tree breaking with tree growth stage, the critical height for tree height of 2, 4, 6, 8 m were selected as 0.5, 1, 2, 2 m respectively, and corresponding the reference diameters are 0.1, 0.124, 0.155 and 0.195 m respectively based on the field observation. The breaking moment for tree height of 2, 4, 6 and 8 m are 10.7, 5.85, 3.25 and 1.85 kNm, respectively. Figure 5(b) shows the numerical result of the threshold water depth (including the increase of water surface elevation due to reflection by the vegetation) of tree breaking for different tree height with different forest width. The line represents the height of 80% of tree height. This is the minimum height of water depth for tree breaking (Tanaka et al. 2009). The threshold water depth is simulated around the observed breaking line, and it increases with increasing in forest width, mainly due to the decrease of velocity.

The reduction of tsunami energy behind a coastal forest depends on the height of tree due to the effect of vertical configuration and tree diameter. $C_{D,all}$ depends on the height of tree and vertical configuration. Figure 6 shows the reduction rate of tsunami force against incident tsunami water depth, where the forest width was selected of 100 m and tree density of 0.2 trees/m² for each tree height case. The initial incident tsunami water depth for tree breaking for the tree height of 2, 4, 6 and 8 m are of 3.9, 4.2, 4.8 and 5.3 m respectively. The taller tree can be reduced higher tsunami energy than the shorter tree. For each tree height case the reduction rate of tsunami force increases as the incident tsunami water depths is higher than the I.B. values. However, for tree height of 2 and 4 m, the reduction rate of tsunami force increases suddenly, and become almost constant as the incident tsunami water depth increases. This is due to all trees can be broken when water depth is larger that the threshold value and the drag resistance by the remaining short aerial root is smaller than the case of 6 and 8 m-trees.

3.3 Breaking of C. equisetifolia and effect of two layers vegetation

In this section, the effect of young C. equisetifolia trees on tsunami reduction was discussed with considering the bending effect. C. equisetifolia. Observation in Laem Son National park in Thailand (Tanaka et al., 2007) showed that only young C. equisetifolia with the trunk diameter about 0.07 m was bending and inclined under the tsunami height of 5 m. The angle $\varphi$ between the trunk and ground was
observed as 15-45°. The bending tree as well as the inclined angle of trunk would make a role on the reduction of tsunami energy, and therefore the bending effect of *C. equisetifolia* should be discussed.

For the cylinder inclined to the flow direction, the drag coefficient may change. The present study used a simple recommendation formula from Post-Tensioning Institute PTI (Poulin and Larsen 2007) on the relation of drag coefficient against the inclined angle as:

\[ C_D = C_{D0} \sin^3 \varphi \]  

(12)

Where, \( C_{D0} \) is the drag coefficient for the case the flow has direction perpendicular to the trunk, corresponding to \( V_0 \) (velocity component perpendicular to the trunk). The project width of vegetation was also changed and depends on the inclined angle.

Figure 7 shows the reduction rate of tsunami force with incident tsunami water depth for three cases of tree growth stage corresponding to trunk diameters of 0.07, 0.1 and 0.15 m, where the forest width, tree density and inclined angle (if breaking occurs) are kept as 100 m and 0.2 trees/m² and 30°. The initial breaking tsunami water depth for the tree diameter of 0.07, 0.1 and 0.15 m are 4.9(5.5), 6.1(7.3) and 7.8(9.9) respectively. The value in brackets indicates the height of water depth including the reflection by the vegetation. According to the result, *C. equisetifolia* is stronger than *P. odoratissimus* against tsunami action. The tree with diameter of 0.07 and 0.1 m was broken by 5.5 and 7.3 m tsunami height. Tanaka et al. (2007) also found the same trunk diameter can be broken by the tsunami height of 5 m (for 0.07 m trunk diameter at Laem Son National park- Thailand) and 10 m (for 0.1 m trunk diameter at Phra Thong Island-Thailand). For all cases the reduction rate of tsunami force increases suddenly as the...
incident tsunami water depths exceeds the threshold breaking values owing to the reduction in drag resistance after tree breaking. With the same forest conditions \((B_{F}, \gamma)\), \textit{P. odoratissimus} can reduce tsunami energy higher than \textit{C.equisetifolia} due to the complex of aerial root, even if the breakage occurs. However, \textit{P. odoratissimus} is not strong when the tsunami water depth greater than 80% of tree height. On the other hand, young \textit{C. equisetifolia} trees \((b_{ref}=0.15 \text{ m})\) remain intact by high tsunamis. In fact, young \textit{C.equisetifolia} can grow densely in natural coastal area and its growth rate is higher than \textit{P. odoratissimus}. Therefore, the combination of \textit{P. odoratissimus} and young \textit{C. equisetifolia} would be effective to protect tsunami (Tanaka et al., 2007, 2009). The present study recommends combined vegetation of \textit{P. odoratissimus} and \textit{C. equisetifolia} as a green belt to protect coastal area from tsunami hazard considering tree breakage phenomenon.

4 Summary and conclusions

Numerical model for estimating tsunami bending moment on a tree and including tree breakage was developed, and the damage length of vegetation, and reduction of tsunami energy were discussed with tree growth stage of \textit{P. odoratissimus} and \textit{C. equisetifolia}, those are dominant in tropical countries. This study can be summarized as follows:

![Figure 6: Reduction of tsunami force \((F_{\text{max}}^*/F_{\text{max0}}^*)\) with incident tsunami water depth, note that subscript 0 indicates the case of no vegetation](image)

![Figure 7: Reduction rate of tsunami force \((F_{\text{max}}^*/F_{\text{max0}}^*)\) with incident tsunami water depth in the front of forest, note that subscript 0 indicates the case of no vegetation](image)
1. The threshold of water depths for starting tree trunk breakage is increased with increasing the forest width, and the analysis satisfies the field investigation results that the critical breaking tsunami water depth is around 80% of the tree height for *P. odoratissimus*.

2. The reduction in water depth and tsunami force decrease when the tsunami water depth exceeds the critical value for breaking. The previous numerical models that do not include the breaking phenomena have a possibility to overestimate the vegetation effect for reducing tsunami force.

3. *C. equisetifolia* is stronger than *P. odoratissimus* against tsunami action. *P. odoratissimus* can reduce tsunami energy higher than *C. equisetifolia* due to the complex of aerial root structures. The Combined vegetation of *P. odoratissimus* and *C. equisetifolia* can be recommended as a green belt to mitigate tsunami hazard considering tree breakage phenomenon.

References

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