Abstract

The direct displacement based design (DDBD) procedure is well developed and used for designing reinforced concrete moment resisting frame structures, wall structures and bridges. However, there is limited number of studies available on designing steel concentric braced frame (CBF) structures using DDBD approach. Therefore, it is necessary to develop a DDBD procedure for CBF structures. On this regards, this paper proposes a DDBD procedure for steel concentric braced frame structures. The proposed procedure utilises yield displacement shape derived on the basis of tensile yielding of the braces, and equivalent viscous damping equation of the system given as a function of system ductility and non-dimensional slenderness ratio by Wijesundara et al. (2011) for concentric braced steel frame structures. Finally, the performance of four steel CBF structures designed according to the proposed DDBD procedure is studied using nonlinear dynamic response of those structures. The results show that the performance of CBF structures is in good agreement with the design considerations.

Keywords: Direct displacement based design, concentric braced steel frames, equivalent viscous damping coefficient, non dimensional slenderness ratio, ductility
1. Introduction

The direct displacement based design (DDBD) was first introduced by Priestley (1993) and it has been subjected to considerable research attention in Europe, New Zealand, and North America in the intervening years (Priestley, 2003). The procedure is well developed for RC moment resisting frame, wall structures and bridges over the last decade. However, DDBD procedure for steel concentric braced frame (CBF) has not been developed fully and only very limited number of studies has been found in the literatures. Medhekar and Kennedy (2000) have developed a displacement based design procedure for (CBF) structures. However, in that design procedure, the equivalent viscous damping (EVD) coefficient of the equivalent single degree of freedom (SDOF) system is taken as 5% of the critical damping. More recently, the DDBD procedure for CBF structures has been developed by Della Corte and Mazzolani (2008), but in that procedure the reference is made to the Takeda-Thin EVD expression which was developed for reinforced concrete (RC) structures. Goggins and Sullivan (2009) reviewed the apparent EVD of a number of CBF structures, with slender braces, subject to shake table testing. They found that the EVD coefficient should be less than that indicated by the Takeda-Thin model, and argued that there was a need for EVD expressions specific to CBF structures. Moreover, in the DDBD procedure developed by Medhekar and Kennedy (2000), the yield displacement profile has been developed on the basis of the yielding of braces but neglecting the axial deformation of columns. Della Corte and Mazzolani (2008) has derived the yield displacement profile of CBF structure using the bucking state of braces. This approach might be appropriate only for the CBF structures with inverted-V braced frames with flexible beams where two concentric braces are connected at each storey level.

As consequences, this study proposed a new DDBD procedure for steel CBF structures based on the yield displacement profile derived using both the tensile yielding of braces and axial deformations of columns, and the EVD coefficient equation developed by Wijesundara et al. (2011). The proposed procedure has been validated using the performance of four steel CBF structures, which were designed according to the proposed procedure. For the validation purpose, the resultant displacement, drift and storey shear profiles at peak storey displacement from nonlinear dynamic analyses (NDAs) are compared with the corresponding design profiles used in DDBD procedure.

2. Direct Displacement Based Procedure for Steel CBF Structures

The complete DDBD procedure for steel CBF structures is summarised here in three steps: (1) evaluation of the yield displacement profile, (2) selection of the design displacement profile, and (3) the transformation of a MDOF system to an equivalent SDOF system referring the first inelastic mode of response.
2.1 Evaluation of Yield Displacement Profile

The yield displacement profile is developed on the basis of following two assumptions: (1) buckling of the compression braces and yielding of the tension braces at all the storey levels occur simultaneously; and (2) the force-deformation curve of a pair of concentric braces at any storey level of a steel CBF structure is approximated to be bi-linear. As stated in the first assumption, achieving the brace yielding and bucking in tension and compression simultaneously is relatively easy in practice for intermediate or stocky braces, since their axial deformations at yielding and buckling are not significantly different. However, this assumption may not be applicable for slender braces which show significant variation between the axial deformation at yielding and buckling (Archambault et al., 1995 and Shaback, 2001).

On the basis of the assumptions, the lateral displacement at each storey level is basically induced due to storey sway mechanism resulting in the brace elongation in tension and shortening in compression and the rigid body rotation of the storey resulting in the axial deformations of the outer columns in the braced bay as shown in Figure 1 (a) and (b), respectively.

\[ \delta_{bi} = L_{d,i} - L_{ud,i} = \varepsilon_y L_{ad,i} \quad (1) \]

where \( L_{ud,i} \) and \( L_{d,i} \) are the undeformed and deformed lengths of the brace at \( i^{th} \) storey, and \( \varepsilon_y \) is the yield strain of the brace steel material. Based on the deformed geometry shown in Figure 1(a), the deformed length of the brace can be expressed as:

\[ L_{d,i}^2 = \left( \frac{B}{2} + \Delta_{sy,i} \right)^2 + h_i^2 = \left( \varepsilon_y L_{ad,i} + L_{ud,i} \right)^2 \quad (2) \]
where \( \Delta_{s,y,i} \) is the lateral displacement induced by the sway mechanism at yielding of the \( i^{th} \) storey tension brace, \( B \) is the bay width and \( h_i \) is the storey height. Since the terms \((\Delta L_{ad,i})^2\) and \((\varepsilon y_{L,ud,i})^2\) are negligible compared to other terms, Eq. (2) can be rewritten as:

\[
\Delta_{s,y,i} = \frac{2\varepsilon_y L_{ad,i}^2}{B} \left( \frac{\varepsilon_y}{\sin \alpha \cos \alpha} \right) h_i
\]

where \( \alpha \) is the angle of the brace to the horizontal line. Considering the rigid rotation at \( i^{th} \) storey as shown in Figure 1(b), the rigid rotation at yielding of the tension brace, \( \theta_{y,i} \) can be expressed as:

\[
\theta_{y,i} = \frac{\Delta_{v,y,i}}{h_i} = \frac{(\partial h_{i,1} + \partial h_{i,2})}{B} = \frac{\Delta_{v,y,i}}{h_i}
\]

Tension and compression forces developed in outer columns in the braced bay, resulting in brace buckling in compression and yielding in tension, are significantly different to each other for the intermediate and slender braces. However, the gravity loads diminish the difference by decreasing the tension force and conversely increase the compression force. As consequence of that, it is reasonable to assume that the axial elongation and shorting of the outer column in tension and compression, respectively are approximately equal. Thus, Eq. (4) can be rearranged in the following form in Eq. (5).

\[
\Delta_{v,y,i} = \left( \frac{2\partial h_i}{B} \right) h_i = \left( \beta \varepsilon_{yc} h_i \right) \tan \alpha
\]

where \( \varepsilon_{yc} \) is the yield strain of the column steel material and \( \beta \) is the ratio of the design axial force to the yielding force of the column section at \( i^{th} \) storey. Finally, the total interstorey yield displacement at the \( i^{th} \) storey \( \Delta_{y,i} \) is:

\[
\Delta_{y,i} = \Delta_{s,y,i} + \Delta_{v,y,i}
\]

\[
\Delta_{y,i} = \left( \frac{\varepsilon_y}{\sin \alpha \cos \alpha} \right) h_i + \left( \beta \varepsilon_{yc} h_i \right) \tan \alpha
\]

### 2.2 Selection of Design Displacement Profile

The design displacement profile proposed to RC moment resisting frame structures by Priestley et al. (2007) is used in this study. As highlighted in the study by Khatib et al. (1988) and Trembley and Poncet (2003) that the first inelastic mode of vibration of concentrically braces frames (CBFs) indicates higher inter-storey drift concentration at the first storey level similar to what is observed in steel moment resisting frames (SMRFs) when they subject to a severe shaking, even though the drift concentration results in both types of structures due to two different structural actions. In CBFs, the drift concentration occurs due to inelastic buckling while it is due to the inelastic rotation of plastic hinges in SMRFs. Thus, it is reasonable to use Priestley et al. (2007) design displacement profile to steel CBFs. The design displacement
profile is obtained from a normalised inelastic mode shape $\delta_i$, and the displacement of the lowest floor $\Delta_1$ as given in Eq. (7):

$$\Delta_i = \delta_i \left( \frac{\Delta_1}{\delta_1} \right)$$  \quad (7)$$

$$\delta_i = \frac{H_i}{H_n} \quad n \leq 4$$  \quad (8)$$

$$\delta_i = \frac{4}{3} \left( \frac{H_i}{H_n} \right) \left( 1 - \frac{H_i}{4H_n} \right) \quad n \geq 4$$  \quad (9)$$

where the normalised inelastic mode shape depends on the height $H_i$ and roof height $H_n$.

### 2.3 Transformation of MDOF System to SDOF System

In the very first stage of the DDBD design procedure, it is essential to represent a MDOF system by an equivalent SDOF system based on the first inelastic mode of response and the transformation of a MDOF system to an equivalent SDOF system is also based on the following assumptions: (1) MDOF system responds harmonically in the assumed shape; (2) the base shears developed by a MDOF system and its equivalent SDOF system are same; and (3) the work done by the lateral earthquake force on both systems is same.

The equivalent SDOF system properties such as effective mass ($m_{\text{eff}}$), secant stiffness ($k_{\text{eff}}$), EVD coefficient ($\xi_{\text{CBF}}$), equivalent design displacement ($\Delta_d$), and the base shear ($V$) can be expressed as described below on the basis of the above mentioned assumptions.

The design displacement ($\Delta_d$) and yield displacement ($\Delta_{y,\text{eff}}$) of equivalent SDOF system found from the assumption 3 by equating the work done by the lateral forces on the each system, can be expressed as given in Eq. (10) and Eq. (11), respectively.

$$\Delta_d = \frac{\sum_{i=1}^{n} m_i \Delta_i^2}{\sum_{i=1}^{n} m_i \Delta_i}$$  \quad (10)$$

where $m_i$ is the mass at the height $H_i$ associate with displacement $\Delta_i$.

$$\Delta_{y,\text{eff}} = \frac{\sum_{i=1}^{n} m_i \Delta_{y,i}^2}{\sum_{i=1}^{n} m_i \Delta_{y,i}}$$  \quad (11)$$

Considering the assumptions 1 and 2, the effective mass of the SDOF system can be defined as:
The effective height of the equivalent SDOF structure is given by:

\[ m_{\text{eff}} = \frac{\sum_{i=1}^{n} m_i \Delta_i}{\Delta_{\text{eff}}}, \quad (12) \]

The design displacement ductility factor of an equivalent SDOF system is related to the equivalent yield displacement \( \Delta_{\text{y,eff}} \), and the design displacement \( \Delta_d \) as:

\[ \mu = \frac{\Delta_d}{\Delta_{\text{y,eff}}} \quad (14) \]

The EVD coefficient of the equivalent SDOF system proposed by Wijesundara et al. (2011) for CBFs, which can be related to the design displacement ductility demand and the non-dimensional slenderness ratio \( \lambda \), is used. The study by Wijesundara et al. (2011) firstly analyses fifteen different pre-determined single storey CBFs in order to evaluate the EVD coefficient through an area based approach (Jacobsen 1960) which is then corrected for the results of inelastic time history analysis (ITHA) conducted using 14 real accelerograms. In general, the area based hysteretic EVD is greatly a function of the non-dimensional slenderness and the ductility, but does not vary significantly for different diagonal bracings. The corrected hysteretic EVD values also exhibit a significant dependency on the non-dimensional slenderness. Therefore, Wijesundara et al. (2011) proposed bilinear damping expressions for design of CBFs as a function of the ductility and the non dimensional slenderness as shown in Eq. (15.a) and Eq. (15.b).

\[ \xi_{\text{CBF}} = 0.03 + \left( 0.23 - \frac{\lambda}{15} \right) (\mu - 1) \quad \mu \leq 2 \quad (15.a) \]

\[ \xi_{\text{CBF}} = 0.03 + \left( 0.23 - \frac{\lambda}{15} \right) \quad \mu \geq 2 \quad (15.b) \]

It is important to note that the non-dimensional slenderness ratio of the equivalent SDOF system is assumed to be equal to the average value of non-dimensional slenderness ratios of the MDOF system. Since, non-dimensional slenderness ratio of the equivalent SDOF system is not available at the beginning of the design procedure, it is suggested to assume a value and verify it after sizing the braces. If the difference between the assumed non-dimensional slenderness ratio and average non-dimensional slenderness ratio is significant, then the design procedure has to be repeated with the new average value of non-dimensional slenderness ratio until the difference is insignificant.
The effective period $T_{eff}$, at the design displacement of equivalent SDOF system is read from the displacement spectrum at the EVD coefficient calculated using either Eq. (15.a) or (15.b).

The secant stiffness corresponding to the design displacement of the equivalent SDOF structure is given by:

$$K_{eff} = \frac{4\pi^2 m_e}{T_e^2}$$  \hspace{1cm} (16)

The design base shear force for the MDOF structure is obtained from the equivalent of SDOF structure as:

$$F_b = K_e \Delta_d$$  \hspace{1cm} (17)

All the steps in the DDBD procedure for steel CBF structures are summarized in the flow chart shown in Figure 3.

3. Nonlinear Dynamic Analysis (NDA)

Four steel CBF structures are designed according to the DDBD procedure described in section 3. Two CBF structures are four and eight storeys with inverted-V bracing configuration and continuous middle column that links the brace-to-beam intersection points at each floor level directly to the foundation while other two frames are four and eight storeys with X bracing configuration. From this point onwards, the inverted-V bracing configuration with middle column is called as IVMC bracing configuration. The floor plan and the elevation of the buildings were predetermined, as shown in Figure 3. The height of each storey is 3.5m and the bay width of each of braced and unbraced bays is 7m. The locations of the concentric braced frames are shown by the bold lines in Figure 2.

![Figure 2: Plan view and elevation of CBF structures with (a) IVMC (b) X bracing configuration](image)

Seven real accelerograms are selected from PEER data base in order to carry out the nonlinear dynamic analyses and scaled to match with the design displacement spectrum of 5% damping as specified in EC-8 (2005), for the soil class A with the PGA of 0.3g. Figure 3 shows the 5% damped displacement spectra of the individual accelerogram and average of the individual
accelerogram together with the design displacement spectrum, in the period range of 0 to 4s. The average displacement spectrum of the individual accelerogram is matched well with the design displacement spectrum.

![Graph](image-url)

**Figure 3: Displacement spectra from the scaled natural accelerogram at 5% damping**

In order to investigate the performance of the building models designed according to the DDBD procedure proposed, nonlinear dynamic analyses are performed using the OpenSEES finite element computer program (McKenna et al. 2007). The steel CBFs are modelled in 3-D rather than in 2-D to permit the braces to buckle in the out-of-plane direction of the frame since all the braces are designed and detailed to develop the out-of-plane buckling. Tremblay et al. (1995) and Tremblay et al. (1996) pointed out that the out-of-plane buckling response of a brace in concentric configuration can occur during a severe shaking. The behaviour of all the frame elements except the braces is limited to in-plane displacement by restraining the translational degree of freedom in the perpendicular direction to the plane of the frame and the rotational degrees of freedom in the out-plane directions. The column-to-base and the beam-to-column connections are modelled as pinned connections while the columns are modelled as continuous members. All the braces are modelled using the inelastic beam-column brace model proposed by Uriz (2005) and Uriz et al. (2008). In this model, each brace is modelled using two nonlinear beam-column elements with five integration points. All the columns and beams are also modelled using nonlinear beam-column elements available in OpenSEES frame work. The corotational theory was used to represent the moderate to large deformation effects of inelastic buckling of braces. Newmark acceleration time integration scheme with beta and gamma 0.25 and 0.5, respectively and tangent stiffness proportional damping equal to 3% of critical damping is adopted for the analyses.

### 4. Results and Discussion

The average profiles of peak displacements, inter-storey drift ratios and storey shears at the peak storey displacement of two different concentric bracing configurations resulting from NDA are compared with the corresponding design profiles.
Figure 4 and 5 illustrate the average displacement and drift ratio profiles for 4 and 8 storeys CBFs with IVMC configuration, respectively. Figure 5 shows clearly that the resultant average displacement profile of 4 storey frame is almost linear and well matched to the design displacement profile. The average drift ratio is 4% below the design drift ratio at the 1st storey while 30% below at the top storey. In the case of 8 storey frame, the average displacement profile is fairly matched with the design displacement profiles ensuring that average displacements do not exceed the design displacements corresponding to the presumed displacement shape significantly as shown in Figure 5. The average drifts at storey levels 5, 6 and 7 are slightly higher than the design drifts, and the maximum of 16% higher average drift is observed at 7th storey level.

**Figure 4: Average time-history response of 4 storey braced frame with IVMC configuration**

**Figure 5: Average time-history response of 8 storey braced frame with IVMC configuration**

Figure 6 shows the average storey shear distribution at the anticipated peak displacements and the design storey shear distribution at the design displacements of 4 and 8 storey frames with IVMC bracing configuration. At the peak storey displacement of any storey level, compression and tension braces are yielded and buckled in tension and compression, respectively. Therefore, the storey shear forces obtained represent the storey shear capacity. From the comparison of design storey shear and the storey shear capacity, it can be concluded that a CBF structure designed according to the DDBD procedure has higher storey shear capacity than the design.
storey shear. It further indicates that the CBF structure could even behave satisfactorily for higher level of shaking than the design level of shaking. Therefore, DDBD procedure results conservative design.

**Figure 6: Storey shear distributions of 4 and 8 storey frames with IVMC configuration**

Figure 7 and 8 illustrate the average displacement and drift profiles obtained from 4 and 8 storey CBFs with X configuration, respectively. Figure 7 exhibits that the resultant average displacement profile slightly deviates from the presumed linear displacement shape resulting a higher drift concentration at the 1st storey. The average drift ratio is 17% above the design drift ratio at the 1st storey while it is 80% below at the top storey. However, exceedance of the drift at the 1st storey is so significant. Similar to the 8 storey frame with IVMC configurations, the average displacement profile of 8 storey frame with X configuration is fairly matched with the design displacement profiles ensuring that average drifts do not exceed the design drifts corresponding to the presumed displacement shape significantly.

Similar to the IVMC configuration, Figure 9 exhibits that the resultant average storey shear forces, at lower storeys are significantly higher than the design values while they are more similar to the design values at the upper storeys.

**Figure 7: Average time-history response of 4 storey braced frame with X configuration**
Current force based design philosophy implemented in many leading codes for CBFs cannot take into account the effects of the slenderness ratio in the response of the structure. It is because the force reduction factors specify for CBFs are independent of slenderness ratio. Therefore, in this study, a DDBD procedure is developed to design steel CBF structures including the effects of slenderness ratio.

The yield displacement profile is evaluated on the basis of tensile yielding of braces. The procedure uses assumed first mode displacement shape proposed by Preistley et al. (2007) for MRFs as the design displacement profile with the EVD equation proposed by Wijesundara et al. (2011). The proposed procedure is validated using NDA results of 4 steel CBFs.

The results of NDA prove that presumed linear displacement shape proposed by Preistley et al. (2007) for low-rise MRFs is reasonably valid for the low-rise CBF structures, even though it depends slightly on the type of concentric configuration. Furthermore, the NDA results of the medium-rise CBF structures also prove that the presumed inelastic first mode displacement shape with higher drift concentrations at lower storeys is a reasonably good estimation for the
displacement shape for medium-rise CBF structures. Unlike the low rise CBF structures, their displacement profiles are less sensitive to the type of concentric configuration.

References


Whitmore, R. E. (1952) “Experimental investigation of stresses in gusset plates,” Bulletin No. 16, Engineering Experiment Station, Univ. of Tennessee, Knoxville, May.
